Maulana Abul Kalam Azad University of Technology, West Bengal

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MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BSM-101

PUID : 01004 (To be mentioned in the main answer script)

MATHEMATICS – 1A

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is

a) Symmetric b) Skew-symmetric

c) Singular d) Orthogonal.

ii) In the MVT $f(h) = f(0) + hf'(\theta h)$, $0 < \theta < 1$, if $f(x) = \dfrac{1}{1+x}$ and $h = 3$ then the value of θ is

a) 1 b) $\dfrac{1}{3}$

c) $\dfrac{1}{\sqrt{2}}$ d) none of these.

[Turn over

iii) The value of the integral $\int_{-\infty}^{\infty} x e^{-x^2} dx$ is

a) $2\sqrt{\pi}$

b) $-2\sqrt{\pi}$

c) $\frac{\sqrt{\pi}}{2}$

d) 0.

iv) If $T : V \to W$ is a linear transformation then Nullity of T + Rank of T equals to

a) dimension of V

b) dimension of W

c) dimension of $V + W$

d) none of these.

v) The sum of the eigenvalues of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 3 & 0 & -3 \end{pmatrix}$ is

a) 1

b) 5

c) 4

d) 0.

vi) If A is a skew-symmetric matrix then $P^T AP$ is

a) Symmetric

b) Skew-symmetric

c) Symmetric and Skew-symmetric

d) none of these.

vii) For which of the following function Rolle's theorem is *not* applicable ?

a) $f(x) = x^2 - 5x + 6$ in [2, 3]

b) $f(x) = \sin x$ in $[-\pi, \pi]$

c) $f(x) = \cos x$ in $[-\pi, \pi]$

d) $f(x) = \cos\left(\frac{1}{x}\right)$ in $[-\pi, \pi]$.

viii) If $\sin x = x - \frac{x^3}{\lambda} + \frac{x^5}{\mu} - \frac{x^7}{5040} + \ldots$ then value of λ and μ are

a) 6 and 120 b) 60 and 1200

c) 600 and 1200 d) none of these.

ix) $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) =$

a) $\frac{2\pi}{\sqrt{3}}$ b) $\frac{2\pi}{3}$

c) $\frac{\sqrt{3}\pi}{2}$ d) $\frac{3\pi}{2}$.

x) The value of 'a' for which $\lim_{x \to 0} \frac{2x + a \sin 2x}{x^2}$ exists finitely is

a) 1 b) -1

c) 2 d) -2.

xi) If $\vec{F} = y^2 z\,\vec{i} + z^2 x\,\vec{j} + x^2 y\,\vec{k}$, then $\vec{F}$ is

a) Irrotational b) Solenoidal

c) both (a) and (b) d) none of these.

xii) The value of the paraboloid generated by revolving the part of the parabola $x^2 = 4ay$, $a > 0$ between the ordinates $y = 0$ and $y = a$ about its axis is

a) $2\pi a^3$ cubic units

b) $4\pi a^3$ cubic units

c) $\frac{4}{3}\pi a^3$ cubic units

d) $\frac{8}{3}\pi a^3$ cubic units.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Use Laplace's expansion to prove that

$$\begin{vmatrix} x & y & z & w \\ -y & x & w & -z \\ -z & -w & x & y \\ -w & z & -y & x \end{vmatrix} = (x^2 + y^2 + z^2 + w^2)^2 .$$

3. Using Lagrange's Mean Value Theorem prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$.

4. Solve the following system of equations by Gauss Jordan's method :

$2x - y - z = 0, \; x + 2y - z = 2, \; 3x - y - z = 1.$

5. Prove that the vectors (1, 0, 1), (1, 1, 0), (1, −1, 1), (1, 2, −3) are linearly dependent.

6. Prove that the set $S = \{(x, y, z) \in I\!R^3 : x^2 + y^2 = z^2\}$ is not a subspace of $I\!R^3$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}$.

b) Prove that the determinant of every orthogonal matrix is 1 or −1.

c) Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

6 + 3 + 6

8. a) Prove that the transformation $T : I\!R^2 \to I\!R^3$ defined by $T(x, y) = (x - y, x + y, y)$ is a linear transformation from $I\!R^2$ to $I\!R^3$.

b) The line segment $x + y = 1, \; 0 \le y \le 1$ is revolved about y axis to generate a curve. Find the lateral surface area of the cone.

c) Examine the function

$f(x,y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x$ for extrema

and indicate the saddle point, if any. 5 + 5 + 5

9. a) Diagonalize, if possible, the matrix $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

b) Use Cayley-Hamilton theorem to find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

c) Apply Gram-Schmidt process to the vectors (1, 0, 1), (1, 0, −1), (1, 3, 4) to obtain an orthogonal basis for $I\!R^3$ with the standard inner product. 6 + 4 + 5

10. a) Prove that $\int_0^{\pi/2} \sin^p x\,dx \times \int_0^{\pi/2} \sin^{p-1} x\,dx = \frac{\pi}{2(p+1)}$.

b) Using Mean Value Theorem prove that $0 < \frac{1}{x}\log\frac{e^x - 1}{x} < 1$.

c) Let T be a linear transformation of $I\!R^2$ into itself that maps (1, 1) to (−2, 3) and (1, −1) to (4, 5). Determine the matrix representation of T with respect to the basis { (1, 0), (0, 1) }. 5 + 5 + 5

11. a) Find the real value of z for which the rank of the matrix $\begin{pmatrix} 1+z & 2 & 3 & 4 \\ 1 & 2+z & 3 & 4 \\ 1 & 2 & 3+z & 4 \\ 1 & 2 & 3 & 4+z \end{pmatrix}$ is less than 4.

b) Find the basis and dimension of the Subspace W of $I\!R^3$ where $W = \{(x, y, z) : x + 2y + z = 0$ and $2x + y + 3z = 0\}$.

c) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, find the value of 'a' and the limit.

5 + 5 + 5

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Paper Code : BS-M-102

PUID : 01035 (To be mentioned in the main answer script)

MATHEMATICS-IB

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$ is

a) $\frac{7\pi}{32}$ b) $\frac{7\pi}{16}$

c) $\frac{5\pi}{32}$ d) $\frac{5\pi}{16}$.

ii) The value of $\Gamma(3)$ is

a) 6 b) 2

c) 24 d) 1.

iii) The singularity of the integral $\int_{-1}^{2} \frac{dx}{x(x-1)}$ are

a) 1, 2
b) – 1, 2
c) 0, 1
d) 0, 2.

iv) The locus of the centre of curvature is called

a) envelope
b) evolute
c) circle of curvature
d) involutes.

v) Which of the following functions does not satisfy Rolle's theorem in [– 1, 1] ?

a) x^2
b) $\frac{1}{x^4+2}$
c) $\frac{1}{x}$
d) $\sqrt{x^2+3}$.

vi) The value of $\lim_{x \to 0} \frac{\sin^2 x}{x^2}$ is

a) – 1
b) 1
c) 2
d) does not exist.

vii) All eigenvalues of any nilpotent matrix are

a) 0
b) 1
c) 2
d) none of these.

viii) If $f(x, y) = \frac{x^2 + y^2}{\sqrt{x+y}}$ then $xf_x + yf_y =$

a) $\frac{1}{2}$

b) $\frac{1}{2}f$

c) $\frac{3}{2}f$

d) none of these.

ix) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

a) $p \geq 1$

b) $p > 1$

c) $p < 1$

d) $p \leq 1$.

x) The value of the determinant $\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix}$ is

a) 2

b) 0

c) 405

d) -1.

xi) If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

a) r

b) 1

c) $\frac{1}{r}$

d) none of these.

xii) The value of t for which

$\vec{f} = (x+3y)\hat{i} + (y-2x)\hat{j} + (x+tz)\hat{k}$ is solenoidal is

a) 2

b) -2

c) 0

d) 1.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $\int_0^\infty \frac{dx}{(1+x^2)^5} = \frac{35\pi}{256}$.

3. The circle $x^2 + y^2 = a^2$ is revolved about the x-axis. Show that the surface area and the volume of the sphere thus generated are respectively $4\pi a^2$ and $\frac{4}{3}\pi a^3$.

4. Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

5. Find the maximum value of $x^3 y^2$ subject to the constraint $x + y = 1$, using the method of Lagrange's multiplier.

6. If $f = x^2 y + 2xy + z^2$, then show that *curl grad* $f = 0$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Expanding the determinant by Laplace's method in terms of minors of 2nd order formed from the first two, prove that

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2 .$$

b) Find the eigenvalues and the eigenvectors corresponding to the smallest eigenvalue of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

c) Check the consistency of the given system of equations and solve if possible :

$x + 2y - z = 10; \quad x - y - 2z = -2; \quad 2x + y - 3z = 8.$

8\. a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - 3 = z$ at the point $(2, -1, 2)$.

b) Check the convergence of the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^2}{2^2} - \frac{3}{2}\right)^{-2} + \left(\frac{4^2}{3^2} - \frac{4}{3}\right)^{-3} + \dots$.

c) Use Mean-Value theorem to prove the following inequality $\frac{x}{1+x} < \log(1+x) < x$, if $x > 0$. 5 + 5 + 5

9. a) Show that the rectangle of maximum area that can be inscribed in circle is a square.

b) Check whether the matrix $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ is diagonalizable or not.

c) Using Parseval's identity corresponding to the Half-Range cosine series of the function $f(x) = x$, $0 < x < 2$, find the sum of the series

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots .$$ 5 + 5 + 5

10 a) Find the Fourier series of the function

$$f(x) = \begin{cases} \pi + 2x, & -\pi < x < 0 \\ \pi - 2x, & 0 \le x \le \pi \end{cases}$$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. 8

b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 7

11. a) If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0 .$$

b) If $y = \tan^{-1} x$ then prove that

i) $(1 + x^2) y_1 = 1$

ii) $(1 + x^2) y_{n+1} + 2nxy_n + n(n-1) y_{n-1} = 0$.

c) Find the directional derivative of $f = xyz$ at (1, 1, 1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.