DR. B. C. ROY ENGINEERING COLLEGE, DURGAPUR

REPORT

CA- 2

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SUBJECT: PHYSICS.

SUBJECT CODE: BS-PH101

TOPIC:

I. Application of Gradient, Divergence and Curl in engineering.
II. Assumptions of Plank's radiation law hence derive the Plank's radiation law.

ROLL NO: 2203014

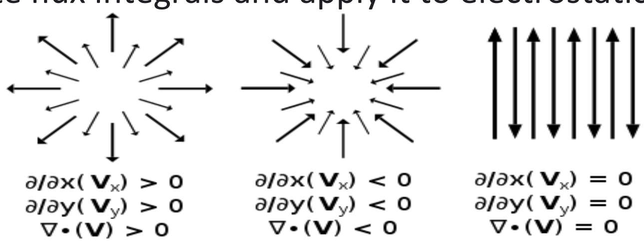
I. APPLICATION OF GRADIENT, DIVERGENCE AND CURL IN ENGINEERING.

Ans.

INTRODUCTION: -

The gradient operator is an important and useful tool in electromagnetic theory. Here's the main idea:
The *gradient* of a scalar field is a vector that points in the direction in which the field is most rapidly increasing, with the scalar part equal to the rate of change. The easiest way to visualize this is with a two-dimensional gradient of concentration (a scalar) in a horizontal plane as in the figure below. In this figure, higher concentrations are shaded more darkly and the blue arrows show the concentration gradient vector field.

The divergence theorem has many uses in physics and engineering; in particular, the divergence theorem is **used in the field of partial differential equations to derive equations modelling heat flow and conservation of mass**. We use the theorem to calculate flux integrals and apply it to electrostatic fields.



(i) **(ii)** **(iii)**

(i) positive divergence **(ii) negative divergence** **(iii) zero divergence**

The name "curl" was first suggested by James Clerk Maxwell in 1871 but the concept was apparently first used in the construction of an optical field theory by James MacCullagh in 1839.
In vector calculus, the **curl** is a vector operator that describes the infinitesimal circulation of a vector field in three-dimensional Euclidean space. The curl at a point in the field is represented by a vector whose length and direction denote the magnitude and axis of the maximum circulation. The curl of a field is formally defined as the circulation density at each point of the field. A vector field whose curl is zero is called irrotational.

METHODOLOGY: -

- **GRADIANT: -**

ψ (x, y, z)

$$\frac{\delta\psi}{\delta x}, \frac{\delta\psi}{\delta y}, \frac{\delta\psi}{\delta z}$$

∴ the gradient of scalar point function ψ (x, y, z) is defined as –

$$\vec{\Delta}\psi = i\frac{\delta\psi}{\delta x} + j\frac{\delta\psi}{\delta y} + k\frac{\delta\psi}{\delta z}$$

The $\vec{\Delta}$ is a vector operator. When its operators on a scalar point

function, it converts the scalar function into a vector function.

$$d\psi = \vec{\Delta}\psi . d\ r\ vector$$

- **DIVERGENCE: -**

Divergence of a vector field at any point is defined as the net outflow or flux of that field per unit volume.

Div vector A = $\Delta\ vector$. A vector = $\left(\frac{\delta}{\delta x}A_X + \frac{\delta}{\delta y}A_Y + \frac{\delta}{\delta z}A_Z\right)$

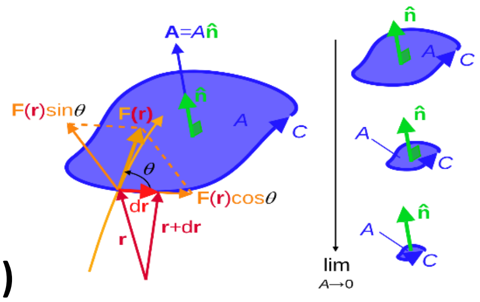
$\Delta\ vector$. A vector is the measure of how much the vector A diverges /Spades out from the point.

- **CURL: -**

The curl of a vector field **F**, denoted by curl **F**, or ∇ × **F**, or rot **F**, is an operator that maps continuously differentiable functions $f : \mathbf{R}^3 \to \mathbf{R}^3$ to continuous functions $g : \mathbf{R}^3 \to \mathbf{R}^3$, and in particular, it maps C^k functions in $\mathbf{R}^3$ to C^{k-1} functions in $\mathbf{R}^3$. It can be defined in several ways, to be mentioned below:

One way to define the curl of a vector field at a point is implicitly through its projections onto various axes passing through the point: if $\hat{U}$ is any unit vector, the projection of the curl of **F** onto $\hat{U}$ may be defined to be the limiting value of a closed line integral in a plane orthogonal to divided $\hat{U}$ by the area enclosed, as the path of integration is contracted indefinitely around the point.

The components of **F** at position **r**, normal and tangent to a closed curve C in a plane, enclosing a planar vector area .



Curl of a vector field (Rotation/ Circulation)

Curl of a vector field measures the rate of reaction of that vector. It is also known as circulation.

$$\because \ \text{curl } \vec{A} = \vec{\nabla} \times \vec{A}$$

$$\Rightarrow \text{curl } \vec{A} = \left(\hat{i}\frac{\delta}{\delta x} + \hat{j}\frac{\delta}{\delta y} + \hat{k}\frac{\delta}{\delta z}\right) \times \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\delta}{\delta y}A_z - \frac{\delta}{\delta z}A_y\right) - \hat{j}\left(\frac{\delta}{\delta x}A_z - \frac{\delta}{\delta z}A_x\right) + \hat{k}\left(\frac{\delta}{\delta x}A_y - \frac{\delta}{\delta y}A_x\right)$$

If $\vec{\nabla} \times \vec{A} = 0$; $\vec{A}$ = irrotational Vector.

RESULTS: -

Some examples regarding gradient: ------

1. The gradient of a line inclined at an angle of 45^0 is-
   m= $\tan 45^0$= 1m= tan450 =1
2. The gradient of a line joining the points (0, 3) and
   (1, 5) is m=5−31−0=21=2m=5−31−0=21=2
3. The gradient of the tangent to the curve y=x2+x+3y=x2+x+3 is ddx.
   y=ddx.
   (x2+x+3) ddx. y=ddx. (x2+x+3) =2x+1+0=2x+1

Some examples regarding gradient: ------

As a result of the divergence theorem, a host of physical laws can be written in both a differential form (where one quantity is the divergence of another) and an integral form (where the flux of one quantity through a closed surface is equal to another quantity). Three examples are Gauss's law (in electrostatics), Gauss's law for magnetism, and Gauss's law for gravity.

The vector field

$$\mathbf{F}(x, y, z) = y\hat{\imath} - x\hat{\jmath}$$

can be decomposed as

$$F_x = y, F_y = -x, F_z = 0.$$

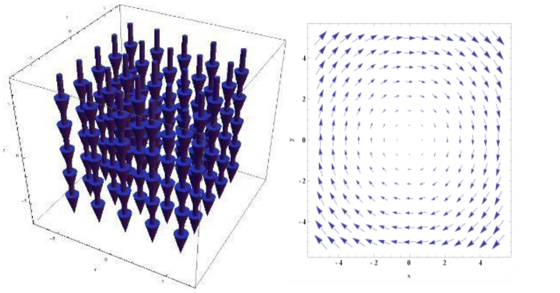
Upon visual inspection, the field can be described as "rotating". If the vectors of the field were to represent a linear force acting on objects present at that point, and an object were to be placed inside the field, the object would start to rotate clockwise around itself. This is true regardless of where the object is placed.

Calculating the curl:

$$\nabla \times \mathbf{F} = 0\hat{\imath} + 0\hat{\jmath} + \left(\frac{\partial}{\partial x}(-x) - \frac{\partial}{\partial y}y\right)\hat{\boldsymbol{k}} = -2\hat{\boldsymbol{k}}$$

The resulting vector field describing the curl would at all points be pointing in the negative z direction. The results of this equation align with what could have been predicted using the right-hand rule using a right-handed coordinate system. Being a uniform vector field, the object described before would have the same rotational intensity regardless of where it was placed.

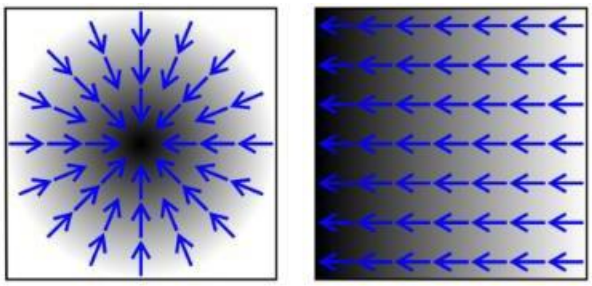
Vector field **F** (x, y) = [y, -x] (RIGHT) and its curl (LIFT).



- In a vector field describing the linear velocities of each part of a rotating disk, the curl has the same value at all points, and this value turns out to be exactly two times the vectorial angular velocity of the disk (oriented as usual by the right-hand rule). More generally, for any flowing mass, the linear velocity vector field at each point of the mass flow has a curl (the vorticity of the flow at that point) equal to exactly two times the *local* vectorial angular velocity of the mass about the point.
- Of the four Maxwell's equations, two—Faraday's law and Ampère's law—can be compactly expressed using curl. Faraday's law states that the curl of an electric field is equal to the opposite of the time rate of change of the magnetic field, while Ampère's law relates the curl of the magnetic field to the current and the time rate of change of the electric field.

DISCUSSION: -

A particularly important application of the gradient is that it relates the electric field intensity E(r)E(r) to the electric potential field V(r)V(r). This is apparent from a review particular view, the battery-charged capacitor example. In that example, it is demonstrated that:



- The *direction* of E(r)E(r) is the direction in which V(r)V(r) decreases most quickly, and
- The *scalar part* of E(r)E(r) is the rate of change of V(r)V(r) in that direction. Note that this is also implied by the units, since V(r)V(r) has units of V whereas E(r)E(r) has units of V/m.

The gradient is the mathematical operation that relates the vector field $E(r)E(r)$ to the scalar field $V(r)V(r)$ and is indicated by the symbol "$\nabla\nabla$" as follows:

$$E(r)=-\nabla V(r)E(r)=-\nabla V(r)$$

or, with the understanding that we are interested in the gradient as a function of position rr, simply

$$E=-\nabla VE=-\nabla V$$

At this point we should note that the gradient is a very general concept, and that we have merely identified one application of the gradient above. In electromagnetics there are many situations in which we seek the gradient $\nabla f\nabla f$ of some scalar field $f(r)f(r)$. Furthermore, we find that other differential operators that are important in electromagnetics can be interpreted in terms of the gradient operator $\nabla\nabla$.

In the Cartesian system:

$$\nabla f = \hat{\mathbf{x}}\frac{\partial f}{\partial x} + \hat{\mathbf{y}}\frac{\partial f}{\partial y} + \hat{\mathbf{z}}\frac{\partial f}{\partial z}$$

In vector calculus, **divergence** is a vector operator that operates on a vector field, producing a scalar field giving the quantity of the vector field's source at each point. More technically, the divergence represents the volume density of the outward flux of a vector field from an infinitesimal volume around a given point.

As an example, consider air as it is heated or cooled. The velocity of the air at each point defines a vector field. While air is heated in a region, it expands in all directions, and thus the velocity field points outward from that region. The divergence of the velocity field in that region would thus have a positive value. While the air is cooled and thus contracting, the divergence of the velocity has a negative value.

Physical interpretation of divergence

In physical terms, the divergence of a vector field is the extent to which the vector field flux behaves like a source at a given point. It is a local measure of its "outgoingness" – the extent to which there are more of the field vectors exiting from an infinitesimal region of space than entering it. A point at which there is zero flux through an enclosing surface has zero divergence.

The divergence of a vector field is often illustrated using the simple example of the velocity field of a fluid, a liquid or gas Thus, the gas velocity has zero divergence everywhere. A field which has zero divergence everywhere is called solenoidal.

Curl **F** is a notation common today to the United States and Americas. In many European countries, particularly in classic scientific literature, the alternative notation rot **F** is traditionally used, which is spelled as "rotor", and comes from the "rate of rotation", which it represents. For avoiding confusion modern authors tend to use the cross product notation with the del operator ∇ × **F**,[2] which also reveals the relation between curl(rotor), divergence, and gradient operators.

Unlike the gradient and divergence, curl as formulated in vector calculus does not generalize simply to other dimensions; some generalizations are possible, but only in three dimensions is the geometrically defined curl of a vector field again a vector field. This deficiency is a direct consequence of the limitations of vector calculus; on the other hand, when expressed as an antisymmetric tensor field via the wedge operator of geometric calculus, the curl generalizes to all dimensions. The unfortunate circumstance is similar to that attending the 3-dimensional cross product, and indeed the connection is reflected in the notation ∇× for the curl.

CONCLUTION: -

The gradient of any line or curve tells us the rate of change of one variable with respect to another. This is a vital concept in all mathematical sciences. As well as the rate of change of distance with respect to time (velocity), there is the rate of change of energy with respect to time (called *power*), the rate of change of chemical concentrations with respect to time (the rate of the reaction), and the rate of change of money owing with respect to time (compound interest). Any system that changes will be described using rates of change that can be visualized as gradients of mathematical functions.

The Divergence of a vector is basically the measure of how much a vector function is spreading out from a particular point in the space. As obvious, divergence of a vector quantity is a scalar. This paper will focus on the importance of divergent thinking in the design process. Divergent thinking is associated with creativity in that multiple unique solutions are generated for a single problem. In contrast, convergent thinking is a process that identifies a single "correct" answer.

The physical significance of Curl: The significance of the curl of a vector field arises in fluid mechanics and in the theory of electromagnetism. In the case of fluid flow, the curl of the velocity field measures the angular velocity of rotation and near the eddy current, it is maximum.

PHYSICAL EXAMPLE: - Electrostatic & Gravitational field.

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I. ASSUMPTIONS OF PLANK'S RADIATION LAW HENCE DERIVE THE PLANK'S RADIATION LAW.

INTRODUCTION: -

In order to explain the distribution of energy in the spectrum of a black body. Max Plank in 1900, put forward the quantum theory of radiation. He assumed that the atoms in the walls of a black-body behave like simple harmonic oscillators, and each has its own characteristics frequency of oscillation. Max Planck discovered a theory that energy is transferred in the form of chunks called quanta, assigned as h. The variable h holds the constant value of 6.63 x 10^{-34} J.s based on the International System of Units, and the variable describes the frequency in s-1. Planck's law helps us calculate the energy of photons when their frequency is known.

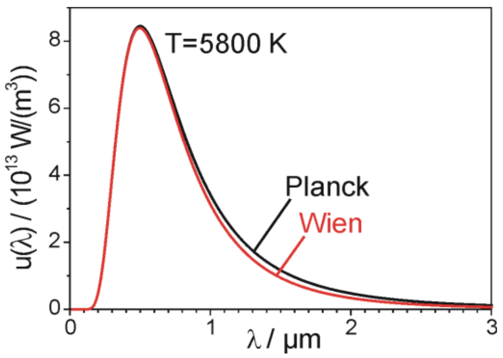
If the wavelength is known, you can calculate the energy using the wave equation to calculate the frequency and then apply Planck's equation to find the energy.

METHODOLOGY: -

Plank's constant: - Plank's constant describes the relevancy between the energy per quantum (photon) of electromagnetic radiation and its frequency. It states that electromagnetic radiation from heated bodies is not emitted as a continuous flow but is made up of discrete units or quanta of energy, the size of which involves a fundamental physical constant (Planck's constant).

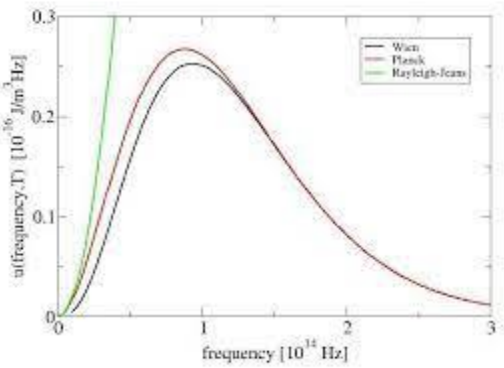
$$h = \text{Planck's Constant} = 6.62 \times 10^{-34}\ Js$$

Plank's Radiation Law in terms of wavelength (λ) is given by –



$$E_\lambda\, d_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{(e^{\frac{hc}{\lambda kT}} - 1)}\, d_\lambda$$

Plank's Radiation Law in terms of frequency (ν) is given by –



$$E_\nu\, d_\nu = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{(e^{\frac{h\nu}{kT}} - 1)}\, d_\nu$$

RESULTS: -

I try to explain this Plank's Radiation law by an example-

Qc. Green light has a wavelength of 525 nm. Determine the energy for the green light in joules.

Solution:

To find the Frequency;

As we know that,

$$c = \lambda \times v$$

$$v = \frac{3\times10^8}{525}$$

Hence,

$$v = 5.71 \times 10^{14}/s$$

To find the energy

As we know that,

$$E = h \times \nu$$
$$(6.626 \times 10^{-34}) \times (5.71 \times 10^{14})$$
$$3.78 \times 10^{-19} J/photon$$

DISCUSSION: -

In order to explain the distribution of energy in the spectrum of a black body, Max Plank the famous scientist, established the quantum theory of radiation i.e., assumed that –

I. **The atoms in the wall of the blackbody behave like simple harmonic oscillator, each having a characteristic frequency of oscillation.**

II. **In thermal equilibrium the emission and absorption of energy by the oscillator occurs at equal rate. The emission and absorption of energy is not continuous but it is in the form of discreate packets of energy know as quantum or photon. The energy emitted by the oscillator is –**

$$E_n = nh\nu \quad (n = 1,2,3 \ldots\ldots\ldots\ldots)$$

III. **The number of oscillators per unit volume within the frequency range ν and (ν+d ν) is given by-**

$$N(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}d\nu \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

IV. According to Maxwell-Boltzmann distribution function the number of oscillators in the energy state En is given by-

$$N_m = N_0 e^{\frac{-E_n}{kT}} \text{.................................. (2)}$$

∴ The energy of a particle vibrating with a frequency ν is-

$$\Rightarrow \langle E \rangle = \frac{\sum_{n=0}^{\infty} N_n E_n}{\sum_{n=0}^{\infty} N_n}$$

$$\Rightarrow \langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu \cdot N_0 e^{-\frac{nh\nu}{KT}}}{\sum_{n=0}^{\infty} N_0 e^{-\frac{nh\nu}{KT}}}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu \sum_{n=0}^{\infty} n e^{-\frac{nh\nu}{KT}}}{\sum_{n=0}^{\infty} e^{-\frac{nh\nu}{KT}}}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu \left[x + 2x^2 + 3x^3 + \cdots\right]}{\left[1 + x + x^2 + x^3 + \cdots\right]}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu x \left(1 + 2x + 3x^2 + \cdots\right)}{\left(1 + x + x^2 + x^3 + \cdots\right)}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu x \, (1-x)^{-2}}{(1-x)^{-1}}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu x}{(1-x)}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu}{\left(\frac{1}{x} - 1\right)}$$

$$\Rightarrow \langle E \rangle = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}} - 1\right)} \quad \text{--- (iii)}$$

∴ Thus, the average energy per degree of freedom is $\left[\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}\right]$ according to quantum theory as against Kt according to the classical equipartition law, Therefore, the energy density of black body radiation between frequency $\boldsymbol{\nu}$ and $(\boldsymbol{\nu}+\mathbf{d}\,\boldsymbol{\nu})$ is, -

$$E_\nu\, d_\nu = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{(e^{\frac{h\nu}{kT}}-1)}\, d_\nu \ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

Therefore, in the terms of wavelength, the energy density in wavelength **$\boldsymbol{\lambda}$ and $(\boldsymbol{\lambda}+\mathbf{d}\,\boldsymbol{\lambda})$ is, -**

$$E_\lambda\, d_\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{(e^{\frac{hc}{\lambda kT}}-1)}\, d_\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

Equation (4), (5) represent Plank's radiation law Which agrees well with experimental energy distribution curve for entire wavelength region of the spectrum.

CONCLUSION: -

Planck's constant talks about the behavior of the particles and the waves on the atomic scale, including the particle aspect of light. Planck's constant is discovery because of the concept that energy can be expressed in discrete units or quantized, this proved fundamental for the development of quantum mechanics. In physics, Planck's law describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T, when there is no net flow of matter or energy between the body and its environment.

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