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A Comprehensive Study on Non-holonomic Kinematic Constraints in Differential Drive Mobile Robots Using Lie Bracket Theory

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Abstract

Differential-drive mobile robots are widely used in autonomous systems but are subject to non-holonomic kinematic constraints that forbid instantaneous sideways motion. This raises the question of whether such platforms can still be driven to arbitrary positions and orientations in the plane. In this work, a kinematic model of a differential-drive robot is analysed using Lie bracket tools to investigate controllability under admissible wheel-velocity inputs, and the resulting distribution is examined in the light of Chow's theorem. To complement the theoretical analysis, numerical simulations are carried out in a physics-based environment that implements the robot's kinematic model with realistic time discretization and control update rates. These simulations show that appropriately sequenced rotational and translational manoeuvres can synthesize effective lateral displacements and steer the robot between different target configurations while obeying its non-holonomic constraint. The combined results clarify how Lie-algebraic structure underpins the manoeuvrability of differential-drive robots and provide concrete motion primitives that can be exploited in motion-planning algorithms for industrial and service robotics.

Keywords: Non-holonomic constraints, Differential drive robots, Lie brackets, Motion planning, Controllability.

1. Introduction

Mobile robots are going increasingly popular and have a wide range of applications in logistics and service industries, social robotics and many other applications like exploration, search and rescue etc. The differential drive robot is one of the fundamental and most widely used platforms among the various types of mobile robot configuration [1]. This is widely used because of its simplicity in design which involves two driven wheels supported by a third non-driven castor wheel for stability.

The Fig. 1 presents the basic principle of motion of differential mobile robots, which lies in controlling the angular velocity of each of the two wheels independently to get a wide range of planar motion without a dedicated steering mechanism in a differentially drive robot. This can be achieved by varying the relative speeds and direction of rotation of these wheels, for example — when both the wheels are rotated at the same speed and in same direction, the bot will move forward in a straight line and if one wheel is rotated in the opposite direction of the other wheel in the same speed, then the bot will rotate about its midpoint of the axle connecting both the wheels.

Kinematic Constraints are certain restrictions that a robot can be subjected to while interacting with environment by its physical embodiment [2]. In context of mobile robots that are operating in a two dimensional plane, kinematic constraints arises from basic assumption such as the wheels rolling without slipping and the absence of lateral movement at the point of contact of the wheels [1]. These can be broadly classified in two categories: holonomic and non-holonomic. Holonomic constraints are represented by algebraic equations that relate the configuration variables of the bot — such as its position and orientation which serve to reduce the system's independent degrees of freedom [3]. For example, a robot that is constrained to move along a pre-set track is of the same category.

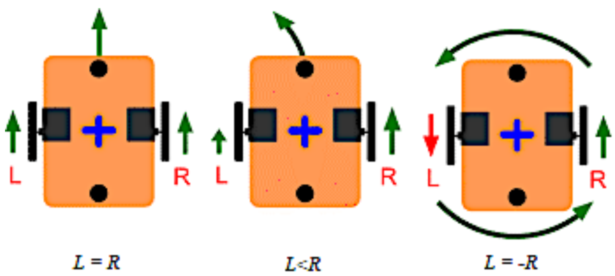

Fig. 1. Basic principle of motions in mobile robots

On the contrary, non-holonomic constraints are expressed as differential equations that involve velocities of the configuration variables. Point to be noted that non-holonomic constraints is that these are non-integrable they cannot be reduced to any algebraic equations in terms of configuration variables alone [3]. These holonomic constraints restrict the bot's instantaneous motion capabilities [4]. An example of a system subjected to non-holonomic constraints is the differential drive robot which has basically limited abilities to move instantaneously in the direction perpendicular to wheels axis i.e. it cannot move directly sideways. This limitation of the differential drive arises directly from the condition where the wheels must roll without slipping and also can't move laterally [1].

This review takes a closer look at the unique movement limitations—called non-holonomic constraints—that are

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