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ORIGINAL PAPER

Study of phasor measurement unit placement in wide area monitoring system of radial distribution network using oppositional-based artificial rabbit optimization

Sneha Sultana[1] · **Sourav Paul**[1] · **Provas Kumar Roy**[2]

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Abstract

In this article, a new metaheuristic bioinspired technique oppositional artificial rabbit optimization (OARO) technique is enhanced and employed, for phasor measurement unit (PMU) placement in distribution system. Based on PMUs, this study emphasizes their advantages for real-time monitoring. Quite differently, the PMU serves as the system's cornerstone. The goal of this kind of technology is to analyse data quickly and automatically arrive at a decision. Four scenarios are taken into consideration in the simulation study for each test system, and the goal is to minimize costs while taking into account fewer PMU. The first case focuses primarily on the number of PMU set as the objective function; the second case uses the wide area monitoring system (WAMS) data traffic index and installation cost to observe comprehensive observability. The third case focuses on how many PMU are used in conjunction with zero injection bus (ZIB). Finally, ZIB and WAMS are implemented together to reduce data traffic and, eventually, the fitness function. There is artificial rabbit optimization (ARO), inspired by the survival of rabbits, such as detour foraging and random hiding. The behaviour of the rabbits in random searching for food in other region neglecting its own region followed by random hiding amongst all borrowings to reduce the chances for the predators to search them to kill. In this study, an oppositional strategy of rabbits' finding the tunnels is applied for finding the food search space. In addition, the rabbit energy shrink strategy is implemented to transmit rabbits from detour foraging and random hiding. The concept of oppositional-based learning applied to rabbit survival strategy has been mathematically modelled and tested in PMU placement problems in the radial distribution system. The methods are evaluated in radial distribution systems with 33, 69, 85, 118, and 141 buses. In each case, it is found that OARO provides better results on PMU placement for phasor measurement of voltage and current in radial distribution systems.

Keywords Radial distribution system · Phasor measurement unit · Artificial rabbit optimization · Oppositional artificial rabbit optimization · Wide area monitoring system · Zero injection bus

1 Introduction

Electricity is a key component of our society. Therefore, a strong supply continuity with the power grid for their consumers is necessary. So researchers concentrated on electricity network operation and monitoring. To monitoring and record the grid status, wide area monitoring system (WAMS) is used in smart grid system [1]. Wide area measurement systems (WAMS) are primarily concerned with gathering real-time synchronized system measurements and distributing them to data-using applications. Phasor measuring unit (PMU) is a tool used to measure electrical parameters such as voltage, current, phasor angle, frequency with time tag in WAMS system. Modern digital signal processors are used in phasor measurement units, which are capable of measuring 50/60Hz AC waveforms (voltages and currents) at a rate of 48 samples per cycle on average [2]. This phasor measuring technique produces very accurate and high-resolution data. PMU is employed in various power nodes under the con-

✉ Sneha Sultana
sneha.sultana@gmail.com

Sourav Paul
sourav.p01@gmail.com

Provas Kumar Roy
roy_provas@yahoo.com

[1] Department of Electrical Engineering, Dr.B C Roy Engineering College, Durgapur, West Bengal, India

[2] Department of Electrical Engineering, Kalyani Government, Kalyani, West Bengal, India

trol of satellite clock for global positioning system (GPS). Recently, PMU received more attention in power system [3]. Estimating the clustering distribution network status is difficult because there are fewer metering devices in distribution networks than in transmission networks. In the multi-area state estimation issue, this topic may result in biased state estimation. In order to account for all the state estimation error components, including estimation bias and estimation error variance, Ghadikolaee et al. [4] suggested a unique multi-objective function for phasor measurement unit placement. In a study introduced by Shankar et al. [5], a brief application of WAMS and PMU was shown. A brief discussion of micro-PMUs, a type of technology capable of monitoring distribution systems, may be found in [6]. In order to enable operators to comprehend the dynamic conditions of the distribution network in real time, Dusabimana et al. created the micro-PMU for distribution network monitoring, diagnostics, and control in [7]. Increased power demand has prompted the need for PMU devices discussed in [8]. PMU system is useful for increase distribution line's working limit, change relay settings and include renewable energy systems [9, 10] presented in [11]. PMU device is also providing backup protection of the power network. PMU is inserted into multiple buses of the system.

So, now a day it becomes a challenging issue for researchers to find the ideal location for PMU installation. Due to a lack of communication infrastructure and inadequate distribution system capabilities, it is neither technically viable nor cost effective to install PMU in all of the power system's nodes. Therefore, in order to install PMU in the substation's best position and number while maintaining complete system monitoring, the binary integer programming (BIP) approach was suggested in paper [12, 13]. In order to solve the optimal PMU placement problem for its computing efficiency, Chen et al. used integer linear programming in [14]. Author Razavi et al. in [15] suggested a linear approach for PMU installation in the paper cite by changing to zero injection buses (ZIB). The main aim of this article was to maximize measurements while using as few PMUs as possible. In this respect, linear integer programming (LIP) was used to solve the optimization problem. Thus, one of the main areas of focus of recent research is on constructing PMUs in an affordable and sensible manner to suit the requirements on system observability. In article [16] the author proposed contingency constrained PMU placement with n-k redundancy criteria as well as zero injection bus (ZIB). The main goals of this article was to reduce the number of PMUs and to increase economic efficiency with the visibility of each bus of the system. This proposed method demonstrated on IEEE 7-bus, 118-bus and 2383-bus network. It is important to limit the number of PMU in each bus to monitor the system is not financially feasible has discussed in [17]. In order to minimize the required number of PMUs, the modified whale optimization algorithm (MWOA) is used in [18] as a solution to the optimal PMU placement problem. Additionally, the suggested approach has been used with the IEEE 14, 30, 57, 118, and 2383 bus systems. Various parameters were presented by Baba et al. [19] to minimize the installed number of PMUs and maximize the network's measurement redundancy. By using component reliability and the analytical hierarchical process (AHP), Kumar et al. in [20] were able to lower the PMU need for full observability under contingency. Author explore topological observability, a novel rule is included that can reduce the quantity of PMUs needed for full system observability. To find the fewest possible PMUs and associated configuration, a modified particle swarm is used as an optimization. A graph-theoretic method is used to provide an initial PMU placement, which is speeds up convergence. Due to the intermittent nature of sources like wind and solar, a real-time precise and faster monitoring apparatus like a phasor measuring unit (PMU) is required to provide information accurately at a faster rate. The author of the work [21] described the implementation of PMU optimal allocation in numerous IEEE test systems by applying the particle swarm optimization (PSO) approach to assure the system's full observability at a standard operating state.

Over the past few years, various researchers have presented distinct WAMS-related works. Many researchers used a variety of optimization methods to address the PMU placement problem. These methods typically offer the finest optimal solution for cost index of PMUs with linear characteristics. However, due to the practical WAMS traffic index's complicated and non-convex features, installing PMUs calls for placing computers, phasor data concentrators (PDCs), and GPS devices inside of secure cabinets. Therefore, these approaches are unlikely to effectively address the issue. Based on the researches, in paper [22] has been delivered regarding PMU placement at suitable node of power system bus. Algorithms are divided into two main groups; heuristic based methods and mathematical based methods. The majority of methods used to determine the best location for PMUs are heuristic based methods, such as simulated annealing (SA), binary search algorithm (BSA), spanning tree algorithm (STA), particle swarm optimization (PSO) are most useful technology for PMU placement presented in references [22–24]. The binary search algorithm was used in the article [25] to obtain the ideal placement of PMU. In article [26] the recursive tabu search (RTS) technique was introduced to find the PMU allocation that will allow for most thorough network monitoring. Researcher Mandava et al. [27] applied STA to obtain the placement of PMUs. With the aim of achieving complete system observability and high measurement superfluity, the optimization problem of PMUs was solved using genetic algorithm (GA) and immunity algorithm (IA) [28, 29]. Using ant colony method, the best location for PMU approach is also taken into account used in [30]. Binary PSO (BPSO) used in article [31] to found the optimum location of PMU. To determine the ideal number and position of PMUs in a distribution network, the author of the paper [32] used BPSO. The author enhanced system monitoring and distribution system state estimation (DSSE) efficiency by utilizing BPSO algorithm. In the paper [33], the author created multi-objective-based emperor penguin optimizer (MOEPO) technique to locate the distribution system's optimal configuration for PMU placement, loss minimization, and voltage profile enhancement. The author used the technique to address the PMU placement issue. The author considered distribution systems with 33 and 69 buses for PMU installations with different distribution network configurations in this article.

According to the literature review, traditional techniques such as MILP, MINLP, and others take more computation time and are less robust. Furthermore, most artificial intelligence-based optimization methods, such as GA, PSO, tabu search (TS), simulated annealing (SA), ant colony optimization (ACO), and differential evolution (DE), suffer from premature convergence. As a result, many papers used hybrid optimization methods to obtain an efficient and reliable solution to the problem by combining their strengths and discarding their weaknesses. When solving a large scale optimization problem, hybrid methods have the disadvantage of being less robust and less computationally efficient. These factors inspired the authors to develop a novel, efficient, simple, and fast population-based optimization technique for solving PMU placement in radial distribution systems.

In the recent past, the concept of oppositional based ARO has been applied in many field of electrical engineering. In [34], the authors placed inverter-based DG installation in the radial distribution system to increase the voltage stability index while minimizing real power, harmonic distortion, and voltage deviation by using OARO. Traditional artificial rabbit optimization (ARO) was presented in [35]. However, by resolving a set of 31 benchmark functions and five engineering issues, the authors of the cited study [35] employed traditional ARO approaches and compared the outcomes with those of other well-known optimizers. The use of phasor measurement unit (PMU) has been facilitated by the intricacy of a power grid with a variety of generation sources. This allows the grid to access data in real time. Digital communication and information flow, however, may be susceptible to cyber-attacks and data injection. Khalid et al. proposed [36] a state estimation based on median regression function (MRF) to tackle this problem. In this present study, the oppositional artificial rabbit optimization (OARO) is used to place PMU in the radial distribution network and find the optimal location. The proposed method is tested on the 33-bus, 69-bus, 85-bus, 118-bus and 141-bus networks with four different scenarios, and the results obtained from OARO have been compared with ARO, particle swarm optimization (PSO) and genetic algorithm (GA). The contributions of this article are summarized as follows:

- The OARO is implemented for cost reduction through the optimal placement of PMU to have system observability.
- The said techniques are implemented in radial distribution systems considering the presence of zero injection bus (ZIB).
- Implementation of wide area monitoring system (WAMS) for PMU to record the status of the radial distribution systems on the grid.
- The said technique is applied in WAMS after the installation of PMU in presence of ZIB.

The rest of the paper is organized as follows. Section 2 presented the problem formulation; Sect. 3 presents the method for solving the PMU placement problem; Sect. 4 presents the proposed optimization techniques based on "oppositional based learning"; Sect. 5 presents the different steps of proposed techniques.; Sect. 6 presents the tests and results of the OARO for the 33-bus and 69-bus radial distribution systems; and Sect. 7 presents the conclusion of the paper.

2 Problem formulation

PMUs are effective in measuring electrical parameters in radial systems in the area of WAMS. PMU measures data from the bus on which it is installed, and neighbouring buses are connected to that bus. According to this hypothesis, a connectivity matrix must be created. The direct measurement of all buses yields similar findings. Due to the lack of faster communication networks, it is practically impossible to install PMUs on all buses, making the endeavour prohibitively expensive. So, the goal is to minimize the amount of PMU to have complete observability of the system while taking the ZIB into account. In addition, the WAMS data traffic index and the installation cost index are considered. The analytical forms are represented as follows.

2.1 Objective function and constraints

2.1.1 Case1: PMU placement for complete observability

PMUs can monitor instantaneous voltages, currents, frequency, etc., at installed points in the power system. Based on the theorems of electrical circuits and measured voltages and currents, installing one PMU at a specific location provides a complete observation state to this location as well as the nearby buses, reducing the total number of PMUs in [37]. The optimal placement of PMUs is formulated by the

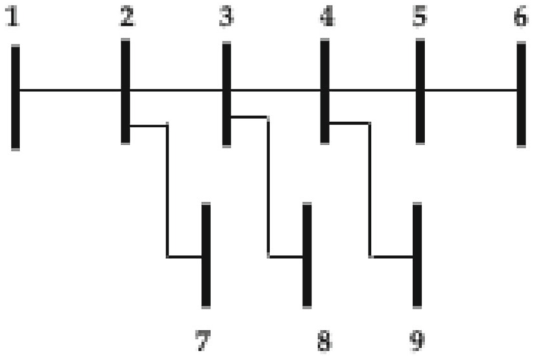


Fig. 1 9-bus distribution diagram

following objective function [12] as stated below in Eq. (1):

$$\text{Minimize } F = \sum_{X=1}^{N} U_X V_X \tag{1}$$

subjected to

$$f(v) = A_{XY} V_X \geqslant Z \tag{2}$$

Here, V_X is a binary decision variable for PMU location whose entries has been defined by Eq. (3):

$$V_X = \begin{cases} 1 & \text{if PMU is present at bus } X \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The connectivity matrix for a N-bus system is represented by A_{XY} as mentioned in (4):

$$A_{XY} = \begin{cases} 1 & \text{if } X = Y \text{ or } X \text{ is connected to} Y \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Equation (1) must satisfy the inequality constraint denoted by Eq. (2). $V_X \epsilon \{0, 1\}$, PMU located at the bus is signified by 1 and no PMU at this bus is identified by 0, U_X is the PMU cost (price), X and Y are bus index numbers, total number of buses are represented by N and Z is the observability constraint, respectively.

For better understanding of constraint, authors have taken a 9-bus system as an example.

As seen in Fig. 1, a network of nine-bus test system may be symbolized by the following constraints which are represented in Eq. (5) and connectivity matrix by Eq. (6), respectively:

$$f(v) = \begin{cases} v_1 + v_2 & \geq 1 \\ v_1 + v_2 + v_3 + v_7 & \geq 1 \\ v_2 + v_3 + v_4 + v_8 & \geq 1 \\ v_3 + v_4 + v_5 + v_9 & \geq 1 \\ v_4 + v_5 + v_6 & \geq 1 \\ v_5 + v_6 & \geq 1 \\ v_2 + v_7 & \geq 1 \\ v_3 + v_8 & \geq 1 \\ v_4 + v_9 & \geq 1 \end{cases} \tag{5}$$

$$A_{XY} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

Because bus 2 is linked to bus 1, the constraints of Eq. (5) demonstrate that PMU must be placed at either bus 1 or bus 2 so that bus 1 is observable. Given that buses 1, 3, and 7 are connected to bus 2, PMU should be installed in any one of the four buses $i.e.$ bus 1, bus 2, bus 3, or bus 7. This will allow bus 2 to be observed. PMUs should be installed on buses 3, 4, 2, or 8 in a similar manner for bus 3's observability, and so on for the other discussed buses. Furthermore, here bus 4 is linked to 3, 5, 9 or with itself in order to make it visible. In a similar way, bus 5 is observable only when PMU is placed on 4, 6 or in itself. For bus 6, it will be either 5 or 6. For bus 7, it will be either 2 or 7. For bus 8, it is either 3 or 8. For bus 9, it is either 4 or 9.

2.1.2 Case2: PMU placement with the existence of ZIB

In many cases of radial distribution networks, ZIB exists to reduce the amount of PMU placements. However, there are some requirements for ZIB identification in distribution networks. They are outlined as below [38]:

(a) There is no load link.

(b) The bus current injection is "0".

(c) Measurements of active and reactive electricity are both zero.

The connectivity matrix, as stated in Eq. (7) will also alter after ZIB identification. The following formula will govern the formation of connectivity:

$$A_{XY} = \begin{cases} 1, & \text{if } X = Y \\ 1, & \text{if bus X and Y are connected} \\ 1, & \text{if bus } X \text{ and } Y \text{ are connected through zero injection bus} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

To understand the significance of ZIB, suppose the ZIB for the specified test system presented in Fig. 1 is bus 2. Afterword, the modification of Eq. (6). Now considering bus 2 as ZIB, the connectivity matrix A_{XY} is represented in Eq. (8).

$$A_{XY} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

2.1.3 Case3: PMU placement followed by WAMS

WAMS is needed in power systems because it allows for the monitoring of system conditions over large areas in order to identify faults and unstable conditions. The main functions of WAMS are to obtain information and to obtain the value derived from it. To prevent regional blackouts events [39], such as those can also be seen in the majority of cases in India, constant monitoring and immediate action are needed if failures occur.

For a PMU at Bus$X$ with Y number of branches, the WAMS data traffic index D_X [40] is represented by Eq. (9) as:.

$$D_X = \sum_{X \in W} S_X C_X^P \quad \mathrm{PF}_{\mathrm{SkBlps}} \tag{9}$$

where all buses with PMU placement is defined by I_X; size of PMU data for Bus$_X$ is represented by S_X; shortest path between Bus$_X$ and control centre is represented by C_X^P. The size of PMU data can be established by Eq. (10) [40].

$$S_X = [(k+1) \times P] \times F_S \tag{10}$$

k = Number of branches for the bus in which PMU is placed(kB). P = size of phasor data unit(kB). F_S = Phasor data frame reporting time. The following guidelines form the foundation of the installation cost index.

- The base installation cost for a PMU linked to a bus is one $p.u.$
- The base installation cost is increased by 0.1 $p.u.$ for each line that is connected to that bus and has additional associated equipment.
- Every bus in a power system needs to follow Rules 1 and 2. Each bus's PMU installation expenses will be included in a row matrix that makes up the resultant vector, or $'I'$.

Installation cost index [40] is represented by Eq. (11):

$$U_X = \sum_{X \in W} I_X \quad p.u. \tag{11}$$

where all buses with PMU placement are defined by W and installation cost matrix for Bus$_X$ is represented by I_X

PMU in all buses is represented by W. The current authors' goal is to lower the installation cost index. The primary goal of this study is to optimize the number of PMUs in a power system while maintaining full observability, minimal WAMS data traffic, and the lowest installation cost. The fitness function of the optimal placement method for complete observability is given for n-bus systems as, fitness function denoted by Eq. (12):

$$\text{Minimize } F = (\alpha + \beta) M_{\mathrm{pmu}} + \beta M_H + \sum_{X \in W} S_X C_X^P + \sum_{X \in W} I_X \tag{12}$$

where the weighted component consists of α and β. In this simulation study, α and β are taken as 1 and 2, respectively, to achieve the most effective values. M_{PMU} is PMU count within the system. The number of buses that are unobserved is N_H.

2.1.4 Case4: PMU placement in WAMS with the existence of ZIB

In power systems, phasor measurement units (PMUs), which are critical components of the wide area monitoring system (WAMS), measure coordinated data at high sample rates while also recording time-stamps. PMU measurements with the same time stamp are used to deduce the state at that moment. The hybridization of WAMS and ZIB further reduces the data traffic index by lowering the fitness function and reducing the amount of PMU in comparison with other mentioned cases.

3 Artificial rabbit optimization (ARO)

Wang et al. have introduced a new optimization technique named ARO reported in [35]. The suggestion of ARO algorithm is described briefly. The fundamental concept, mathematical technique of detour forging and random hiding strategy used by rabbits to survive in earth, are investigated and explained.

Basic idea

The fundamental idea of ARO is taken from existence policy of rabbits in the earth. A short clarification is introduced in the rest. Rabbit has a good survival strategy from all the others human beings. Rabbits are basically phytophagous, belongs in the lower food chain and so there have many exploiters.

To survive from the exploiter, rabbits follow two main survival policy. First one is detour foraging (exploration). According to this strategy, rabbits are try to hide their nest from the exploiters. They do not eat plants near their own nest. Rabbits use a large region to look for food, assisted by overhead scanning or a broad field of vision. This method used in rabbits is termed as detour foraging. The second survival policy of rabbits is random hiding (exploitation). Rabbits make many holes around their own nest, to hide the shelter from hunter. They randomly choose another hole for their own shelter to survive. Rabbits are considered as a primary consumer and small in size. Rabbits have strong and tall back legs, strong muscles, which help to run faster. They quickly reverse and flee in zigzag pattern from the trappers, which essentially increase the likelihood that the rabbits will survive followed by [41]. Due to the escaping nature, they lost their energy; therefore, the rabbits must adjust by changing the strategy between detour foraging and random hiding stage according to the energy level.

Mathematical model of algorithm rabbits alter their foraging and hiding strategies in accordance with their propensity to conserve energy. In ARO, this strategy of rabbits is explained. The suggested ARO is presented, the following description in the mathematical model. As previously stated, in the stage of foraging, rabbits look far food in the area other than their own or in areas that are far from their nest.

3.1 Detour foraging

Detour foraging is the term used to describe this type of foraging in rabbits. Assume each rabbit in ARO swarm has a separate area with some grass in addition to d tunnels, which the rabbits always visit each other's locations at random to forge. Mainly in the stage of foraging rabbits are apt to scurry around food source to find enough food for survive. Because of this, ARO's detour foraging behaviour indicates that each search creature tends to renovate their location in relation to another search creature at random from swarm and add a disruption. The following is the suggested mathematical model as represented by Eq. (13) of rabbit's detour foraging:

$$\vec{u}_a(T+1) = round\Big(\vec{y}_b(T) + S \times (\vec{y}_a(T) - \vec{y}_b(T)) + round(0.5.(0.05 + z_1)) \times m_1\Big) \tag{13}$$

The perturbation in Eq. (13) might help ARO to avoid local extrema and carry out a global search.

$$S = K \times d \tag{14}$$

$$K = \left(e - e^{\left(\frac{T-1}{P}\right)^2}\right) \times \sin 2\pi z_2 \tag{15}$$

$$d(f) = \begin{Bmatrix} 1 & \text{if } f == h(i) \\ 0 & \text{else} \end{Bmatrix} \tag{16}$$

$$h = \text{rand perm } (d) \tag{17}$$

$$m_1 \sim M(0,1) \tag{18}$$

where $\vec{u}_a(T+1)$ is represented as the candidate position of the a th rabbit at the time $T+1$, $\vec{y}_a(T)$ represented as the position of the a th rabbit at the time T, m is represented as size of a rabbit population, d is represented as the dimension of the problem, P is the maximal number of iterations, randperm returns a random permutation of the integers from 1 to d as depicted by Eq. (17) by the variable h, z_1, z_2 and z_3 are three random numbers in (0,1) [35], K is represented as the running length which indicates the movement pace when performing the detour foraging, and m_1 is subjected to the standard normal distribution as depicted by Eq. (18).

The disruption in Eq. (16) may help ARO avoid local extrema and carry out a global search. While in subsequent iterations, a step of this duration might be shorter. K represented by Eq. (15) denotes a bigger step size, which can aid in investigation while a smaller step size aids in exploitation. d is represented a mapping vector that can assist the algorithm in selecting at random how many components of search individuals should change in the foraging behavior. S symbolizes the running operator which is described (Eq. 14) the running characteristic of rabbits.

According to Eq. (16), individuals look for food at random based on where they are in relation to one another. Due to the nature of a rabbit, they can travel a great distance to the areas of the other rabbits. This unique foraging strategy of visiting other people's nests rather than their own remarkably adds to exploration and ensures the ARO algorithm's ability to perform global searches.

3.2 Random hiding

A rabbit typically excavates several distinct tunnels around its nest to use as hiding places when fleeing from hunters. In ARO, a rabbit always digs d tunnels around it along each facets of the search area, and it always picks one burrow at random to hide in to lessen the likelihood of being eaten. In this respect, the following equation is provided.

$$\vec{R}_{a.b}(T) = \vec{y}_a(T) + D \times h \times \vec{y}_a(T), \quad a = 1, \ldots, m, \text{ and } b = 1, \ldots, d \tag{19}$$

$$D = \frac{P - T + 1}{P} \times z_4 \tag{20}$$

$$m_2 \sim M(0,1) \tag{21}$$

$$H(f) = \begin{Bmatrix} 1 & \text{if } f == b \\ & f = 1, .., d \\ 0 & \text{else} \end{Bmatrix} \tag{22}$$

The d tunnels in a rabbit's immediate area are created along each dimension using Eq. (19). Throughout the course of iterations, D, the hiding parameter illustrated in Eq. (20), decreases linearly from 1 to $\frac{1}{T}$ with a random disruption discussed in [35]. These tunnels are originally created in a larger area near a rabbit, in accordance with this specification. This neighbourhood shrinks as the versions get more numerous.

As was already stated, hunters frequently pursue and attack rabbits. Rabbits must locate a secure hiding place if they are to live. In order to prevent being captured, they are forbidden from choosing a tunnel at random from among their tunnels to shelter in. The following equations are suggested to represent this random hiding technique mathematically:

$$\vec{u}_a\left(T+1\right) = round\left(\vec{y}_a(T) + S\left(z_4 \times \vec{R}_{a,z}(T) - \vec{y}_a(T)\right)\right),\ a = 1, .., N \tag{23}$$

$$h_z\left(f\right)\begin{cases} 1 & if\ f == [z_5 \times d] \\ & f = 1, \ldots, d \\ 0 & \text{else} \end{cases} \tag{24}$$

$$\vec{R}_{a.z}\left(T\right) = \vec{y}_a\left(T\right) + D \times h_z \times \vec{y}_a\left(T\right) \tag{25}$$

where $\vec{R}_{a,z}$ is symbolized as a randomly selected tunnel (25) for hiding from its j tunnels, and, z_4 and z_5 are two random values in (0,1). The a th search individual will attempt to update its position in relation to the randomly chosen tunnel from its j tunnels based on Eq. (23). Following the successful completion of either detour foraging or random hiding, the ath rabbit's position is updated as follows:

$$\vec{y}_a(T+1) = \left\{\begin{array}{ll} \text{round}\left(\underset{a}{\vec{y}}\ (T)\right) & \text{fitness}\vec{y}_a(T) \le (\vec{y}_a(T+1)) \\ \vec{u}_a(T+1) & \text{fitness}(\vec{y}_a(T)) > \text{fitness}(\vec{y}_a(T+1)) \end{array}\right\} \tag{26}$$

According to this Eq. (26), the rabbit will leave its current position and remain at the candidate position created by either Eq. (13) or Eq. (23) if the fitness of the candidate position is greater than the fitness of the rabbit.

3.3 Energy reduction (change stage from exploration and exploitation)

Rabbits have a tendency to change their stage between detour foraging and random hiding, according to the rabbit's energy level. In ARO, rabbits always have a tendency to habitually engage in detour foraging during the early iteration and habitually engage in random hiding during the later iterations. This search procedure is powered by a rabbit's energy, which steadily diminishes over time. In order to simulate the transition from exploration to exploitation, an energy factor is created. The following is a definition of the energy component in ARO is shown in Eq. (27):

$$Q\left(T\right) = 4\left(1 - \frac{T}{P}\right)\ln\frac{1}{z} \tag{27}$$

where z represents the random value in (0,1). With the oscillation amplitude during the iterations, the energy factor $Q(T)$ exhibits a tendency towards zero. The high value of the energy factor indicates that a rabbit has enough physical stamina and endurance to engage in detour foraging. A rabbit needs random hiding since it is less physically active, as indicated by the modest value of the energy component. As a result, in ARO, when the energy component $Q(T) > 1$, a rabbit is more likely to forage randomly in other rabbits' territories during the exploration phase, as a result, detour foraging occurs. In the exploitation phase, a rabbit is more likely to arbitrarily exploit its own burrows when the energy factor $Q(T) \le 1$, which leads to random hiding. ARO can alternate between random hiding and detour foraging depending on the energy component Q's value. That is to say, when $Q(T) > 1$, investigation happens, and when $Q(T) \le 1$, exploitation happens. The probability of $Q > 1$ is calculated in order to look into how the energy component affects the algorithm's search ways. Let, $\gamma = 2.\ (1 - \frac{T}{P})$ then $Q(T) = 2.\ln\frac{1}{z}.\gamma$ the probability of $Q > 0$ is calculated by: the iterative procedure, detour foraging has a chance of about 0.5. In other words, the ARO algorithm performs detour foraging and random hiding almost equally in the iterative process, which considerably aids in balancing exploitation and exploration. The emphasis on exploration and exploitation in that order is a crucial factor to consider when assessing an optimization algorithm, and these search behaviors must be presented throughout optimization presented in [42]. The energy factor Q's range, which steadily grows with the number of random oscillator iterations, controls the search behaviors in ARO. On the one hand, the algorithm is compelled by the decreasing tendency of Q to frequently perform exploitation in the later iterations while performing exploration frequently in the earlier iterations. Additionally, it helps the algorithm gradually transition from worldwide to local search. On the other hand, the decreasing tendency of Q, which initially supports exploration and then exploitation, is unaffected by the random oscillator. Even in the concluding iterations of the algorithm, the energy factor Q causes some exploration is calculated in Eq. (28).

$$\begin{aligned} R\left\{Q\left(T\right) > 1\right\} &= \frac{\int_0^2 \int_0^{e^{\frac{-1}{2f}}} \mathrm{d}z\mathrm{d}\gamma}{1.2} \\ &= \frac{1}{4}\int_{\infty}^{\frac{-1}{4}} \frac{e^T}{T}\mathrm{d}T + e^{\frac{-1}{4}} \approx 0.5177 \end{aligned} \tag{28}$$

The ARO algorithm creates a population of artificial rabbits (possible solutions) in the search area at random. Each time, a rabbit iterates, it updates its location in relation to either a randomly selected rabbit from the community or a randomly selected rabbit from one of its tunnels. With more iterations, the energy factor Q decreases, which can drive each member of the population to alternate between detour foraging behaviour and random hiding behavior. The best-so-far answer is returned once all updating and calculation have been completed interactively and the termination criterion has been satisfied.

3.4 Swarm behaviours

A swarm behaviour simulation is used to visually observe the search behaviors of the suggested mathematical models. Thirty rabbits' swarm behaviors are used to find the best possible combination of an objective function. Assume that the global optimum is represented by green points, and the yellow balls stand in for thirty rabbits. These thirty rabbits are first produced at random in the search space and they investigate the whole thing. All of the rabbits progressively reduce their search area and take use of the area surrounding the optimal value as time T increases. All of the rabbits eventually converge on the global optimum.

4 Oppositional based learning

One of the most popular and efficient optimization method known as opposition-based learning (OBL) was created by Tizhoosh [43]. It helps improve the accuracy and speed of convergence of the simulation results. Here, the search space is used to configure the initial solution, which is then updated through iteration. The distance between the initial position of the technique and the ideal position has a significant impact on how quickly it works. Less computing is required for a certain technique to predict the initial values closer to the to the best solution. Therefore, execution time should be as short as possible in order to effectively implement a naturalistic strategy. Therefore, the opposite solution and random solutions are generated simultaneously to reduce the computation duration. Researchers have demonstrated in previous work that ARO is highly flexible in terms of finding the best solution. However, two significant issues conventional ARO faces are slow convergence rate and local optima entrapment. In addition to these, they occasionally have issues with lengthy computation durations. In this work, the concept of is integrated with the original ARO in order to accelerate convergence mobility and overcome the stated limitations of ARO. Researchers using OBL claim that contrary solutions are typically more closely related to the best responses than random responses. In this study, the opposite number is taken into account in the series of iterations to arrive at a nearly global response, not just at the initialization step [44]. In the search area, the oppositional and numbers are utilized to improve the current algorithms. Z_a be a real number defined between $[m_a, n_a]$ depicted in Eq. (29). Reference [44] used the following formula to describe mathematically its opposite number Z_a^0:

$$Z_a = m_a + n_a - Z_a^0 \tag{29}$$

Opposite point is defines as: $J^o = \left(Z_1^o, Z_2^o, ...Z_n^o\right)$ be a space in the search space with (Eq. 30)

$$Z_a \in [m_a, n_a]\ \forall\ a = \left\{0, 1, 2, ...n_j\right\} \tag{30}$$

The opposite point $J^o = \left(Z_1^o, Z_2^o, ...Z_n^o\right)$ is defined by its component:

$$Z_{n,a}^o = m_a + n_a - (m_a + rand(0, 1) \times (n_a - m_a)) \tag{31}$$

Where $a = 1, 2, ..., \text{dim};\quad n = 1, 2, ..., \text{popsize}$

Let $f(x)$ be the corresponding fitness function value derived optimally. Accordingly, J^* $(\text{oz}_1, \text{oz}_2, \ldots .\text{oz}_n)$ is the opposite population of J $(z_1, z_2, \ldots, z_n)$. If $f(J^*) \leq f(J)$, then replace J by J^*, otherwise go with J [44]

Opposition based initialization:

the pseudo-code listed below may be followed:

```
for  $a = 1 : P_S$
    for  $b = 1 : C_v$
            $Z_{a,b}^0 = I_a + J_a - Z_{a,b};$
    end
end
```

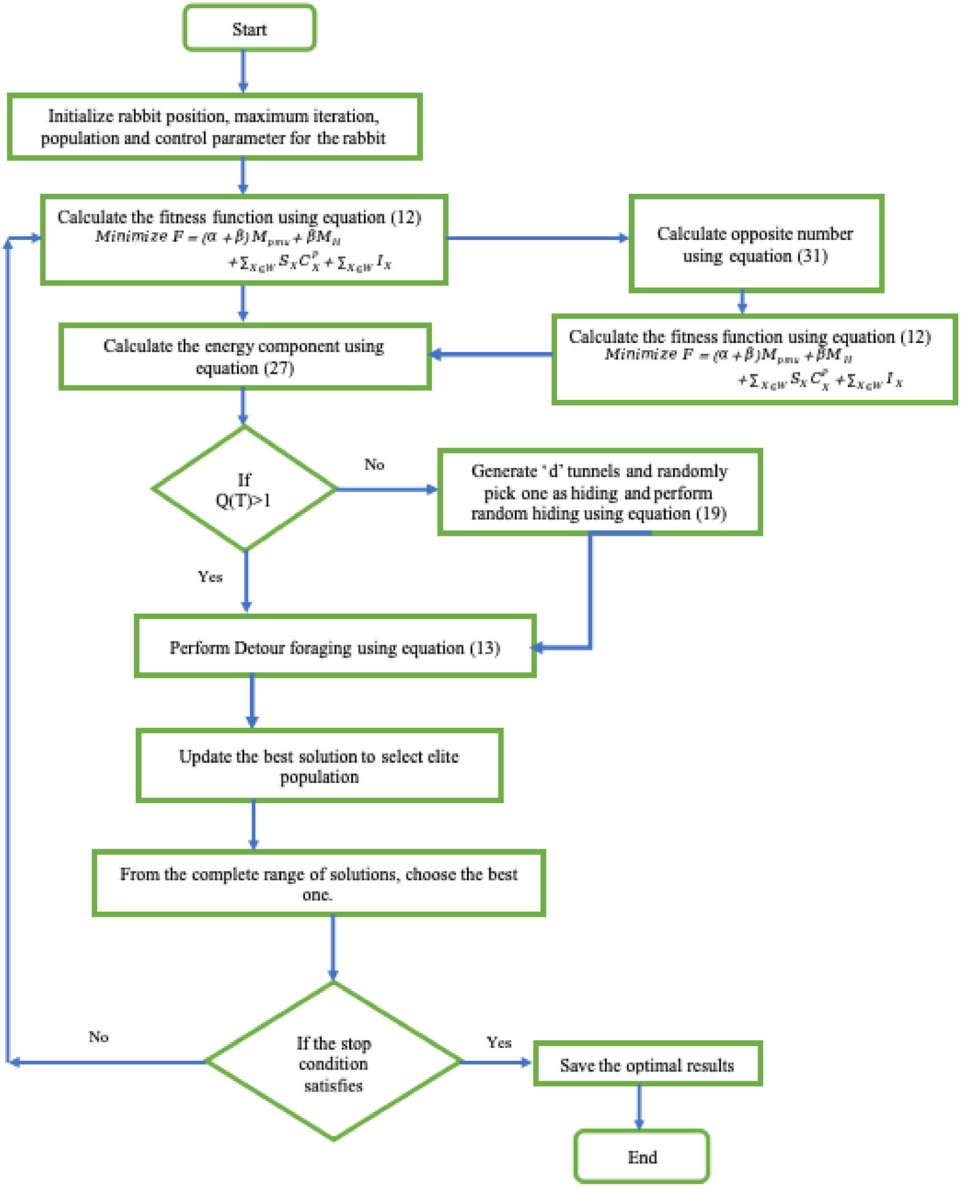
The following actions must be taken when OBL is first initialized [45]:

In the designated space, randomly estimate the first search agents of size $N_p * \text{Dim}$

Determine the opposite values for each population using equation (29)

Choose the best N_J answer and use it as the current population for the following generation.

Fig. 2 Flowchart of proposed OARO to place PMU in radial distribution systems



5 Implementation of OARO for allocation of PMU

One of the requirements in every PMU placement problem in radial system is power flow. Power flow is necessary because limits like the voltage of the nodes and the current of the branches must be respected regardless of whether the problem is addressed mathematically or via meta-heuristic techniques. Therefore, it is necessary to pay special attention to power flow in the distribution network. Another requirement is the solution method. Since almost all the mathematical methods that have been used so far in solving the PMU placement problem are single-objective and used only to find the allocation of PMU in transmission systems, and since in this article, the PMU placement is supposed to be done simultaneously with the presence of WAMS and ZIB, and also want to minimize number of PMU requirement, data traffic index, installation cost and the fitness value. In this article, the OARO is used in radial systems to solve four cases applied on five test systems. The complete flow of the proposed OARO technique is given in Fig. 2. The various steps in the procedure are enumerated below.

Step 1: Create a group of rabbits at random.

In ARO, the control variables of solution vector are randomly build inside the search space. The solution variables for number of PMU (Tie_i), are formed by using the following approach (32):

$$\begin{aligned} \mathrm{PMU}_i = round\big[&\mathrm{PMU}_{\min,i} + rand(0,1) \\ &\times (\mathrm{PMU}_{\max,i} - \mathrm{PMU}_{\min,i}\big];\ i = 1, 2, ..., N \end{aligned} \tag{32}$$

where, $\mathrm{PMU}_{\min,i} = 1$ and $\mathrm{PMU}_{\max,i}$= number of buses.

Afterward, obtain input information such as line impedance, load power, and P centre eight while simultaneously setting the ARO algorithm's control parameters. Equation (27) can be used to compute the energy factor Q for each rabbit individual.

Step 2: Pick PMU at random from the other people if $Q > 1$. Use Eqs. (14)–(18) to calculate S. Utilise Eq. (13) to carry out detour foraging. Using Eq. (26), adjust the current person's location. Inspect the working limitations of the control variables and check whether they are within operating constraints or not. If any variable is lower or higher than its limiting value then fix it to its limiting value. Equations (1), (7), (9), (11) and (12) are used to calculate the fitness value of each solution set.

Step 3: Employing Eq. (19)–(22), do random hiding. Calculate the fitness for various set PMUs and update location using Eq. (26)

Step 4:Create the opposing population by using the following jumping rate R^j description:

```
$if \ \ rand \ (0,1) < R^j$
    $for \ \ a = 1 : P_S$
        $for \ \ b = 1 : C_v$
            $Z^0{}_{a,b} = round\left(I_a + J_a - Z_{a,b}\right)$
        $end$
    $end$
$end$
```

Step 5: Stop if this is the best answer; otherwise, keep going through the stages above until the requirements are satisfied.

6 Result and discussion

The proposed method is implemented on 33-bus, 69-bus, 85-bus, 118-bus and 141-bus radial distribution system in MATLAB software. By utilizing a 64-bit MATLAB R2016b environment, the coding has conducted. Taking into consideration the stochastic nature of meta-heuristic algorithms, each experiment was repeated 100 times. The simulation results of the proposed method also have been compared with PMU placement random solutions and artificial rabbit optimization (ARO) to show its effectiveness in achieving betters results for this purpose. To guarantee a level playing field, all algorithms were executed up to a maximum of 200 iterations. The algorithms were built and run on an optimized laptop, featuring a powerful 1.8 GHz Dual-Core Intel Core i5 processor, 8 GB 1600 MHz DDR3 RAM, APPLE SSD SM0128G storage, and Intel HD Graphics 6000 1536 MB that is run on windows 10 Pro operating system. The goal is to achieve system observability by reducing the volume of WAMS data transmission and lowering the cost of PMU installation. In this section, to evaluate the proposed method, its performance is examined in four scenarios:

- Scenario 1: Optimal PMU placement to have complete observability.
- Scenario 2: Simultaneous installation cost as well as fitness value minimization and PMU placement in WAMS.
- Scenario 3: Placement of PMU in the network with the presence of ZIB.
- Scenario 4: Placement of PMU considering above both cases.

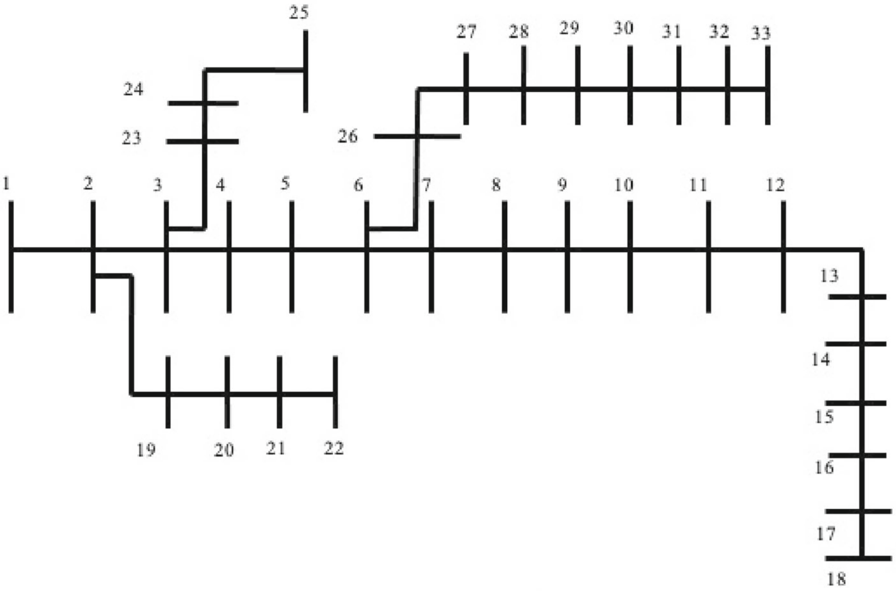
The authors in the present work have proposed OARO in optimal placement of PMU and compared the computed results with the ARO predecessor, the superiority of the suggested techniques has been demonstrated. The two versions are compared using a range of factors, such as convergence characteristics, installation cost, and fitness function. All starting values are set to 50 for the population size and 100 for the maximum number of iterations. Every optimizer's output is based on thirty runs.

6.1 Result analysis

The authors in the present study have analysed a 33-bus system as depicted in system (Fig. 3). Here D_X represented both the connecting bus and the bus with PMU locations. As determined by OARO, the best PMU positions for scenario 2 are [2, 5, 6, 7, 9, 12, 15, 18, 21, 24, 27, 30 and 33]. Therefore, the following values can be used to express the size of PMU data for the aforementioned buses: $S_2 = 4$; $S_5 = 3$; $S_6 = 4$; $S_7 = 3$; $S_9 = 3$; $S_{12} = 3$; $S_{15} = 3$; $S_{18} = 2$; $S_{21} = 3$; $S_{24} = 3$; $S_{27} = 3$; $S_{30} = 3$; $S_{33} = 2$; and also, the shortest path between PMU buses and control centres are $C_2^8 = 6$; $C_5^8 = 3$; $C_6^8 = 2$; $C_7^8 = 1$; $C_9^8 = 1$; $C_{12}^8 = 4$; $C_{15}^8 = 7$; $C_{18}^8 = 10$; $C_{21}^8 = 9$; $C_{24}^8 = 7$; $C_{27}^8 = 4$; $C_{30}^8 = 7$; $C_{33}^8 = 10$; now, Eq. (9) is used to express the WAMS data traffic index. In this example, $D_X = \sum S_X C_X^P$; where $X \in 2, 5, 6, 7, 9, 12, 15, 18, 21, 24, 27, 30, 33$, *i.e.* $D_X = S_2C_2^8 + S_5C_5^8 + S_6C_6^8 + S_7C_7^8 + S_9C_9^8 + S_{12}C_{12}^8 + S_{15}C_{15}^8 + S_{18}C_{18}^8 + S_{21}C_{21}^8 + S_{24}C_{24}^8 + S_{27}C_{27}^8 + S_{30}C_{30}^8 + S_{33}C_{33}^8 = 201 PF_SkBlps$. Next the installation cost index for this particular location can be written as

$U = \sum I_X$ where $X \in 2, 5, 6, 7, 9, 12, 15, 18, 21, 24, 27, 30, 33$ The total installation cost index for scenario 2 is 14.3 *p.u.*, as shown in Table 1. This is because, $I_2 = 1.2$; $I_5 = 1.1$; $I_6 = 1.2$; $I_7 = 1.1$; $I_9 = 1.1$; $I_{12} = 1.1$;

Fig. 3 33-bus distribution diagram



$I_{15} = 1.1$; $I_{18} = 1$; $I_{21} = 1.1$; $I_{24} = 1.1$; $I_{27} = 1.1$; $I_{30} = 1.1$; $I_{33} = 1$; PMU locations for scenario 4 where WAMS is involved in the presence of ZIB are [2, 5, 8, 11 14, 17, 21, 24, 26, 29, 32]. The size of PMU data for the aforementioned buses may therefore be expressed using the values below: $S_2 = 4$; $S_5 = 3$; $S_8 = 3$; $S_{11} = 3$; $S_{14} = 3$; $S_{17} = 3$; $S_{21} = 3$; $S_{24} = 3$; $S_{26} = 3$; $S_{29} = 3$; $S_{32} = 3$; The quickest route between PMU buses and control centres is also $C_2^8 = 6$; $C_5^8 = 3$; $C_8^8 = 1$; $C_{11}^8 = 3$; $C_{14}^8 = 6$; $C_{17}^8 = 9$; $C_{21}^8 = 9$; $C_{24}^8 = 7$; $C_{26}^8 = 3$; $C_{29}^8 = 6$; $C_{32}^8 = 9$; now, the WAMS data traffic index is expressed using Eq. (9). In this instance, $D_X = \sum S_X C_X^P \quad where \quad X \in 2, 5, 8, 11, 14, 17, 21, 24, 26, 29, 32$; $i.e.\ D_X = S_2C_2^8 + S_5C_5^8 + S_8C_8^8 + S_{11}C_{11}^8 + S_{14}C_{14}^8 + S_{17}C_{17}^8 + S_{21}C_{21}^8 + S_{24}C_{24}^8 + S_{26}C_{26}^8 + S_{29}C_{29}^8 + S_{32}C_{32}^8 = 192PF_SkBlps.$ The installation cost index for this specific location can then be expressed as follows: $U = \sum I_x$, where $X \in 2, 5, 8, 11, 14, 17, 21, 24, 26, 29, 32$. According to Table 1, the overall installation cost index for scenario 2 is 12.2 $p.u.$. This is due to, $I_2 = 1.2$; $I_5 = 1.1$; $I_8 = 1.1$; $I_{11} = 1.1$; $I_{14} = 1.1$; $I_{17} = 1.1$; $I_{21} = 1.1$; $I_{24} = 1.1$; $I_{26} = 1.1$; $I_{29} = 1.1$; $I_{32} = 1.1$; the same procedure is also applicable to scenario 1 and to other bus cases.

6.1.1 33-bus studied case

The 33-bus radial distribution system is an established standard that specifies principles and suggestions for electricity systems is shown in Fig. 3. The system data and necessary network configuration of the considered test system are available in [46]. Table 1 depicts the resulting combination of minimal numbers of PMU for the four scenarios as discussed above. The meta-heuristic techniques generate multiple combinations that satisfy the overall system observability.

Table 1 shows several combinations with a minimum number of PMUs using the proposed OARO for total observability, as well as their WAMS data traffic index and installation cost index values. As a result of the reduced data traffic and installation costs, a better solution is created. PMU placement on WAMS is being used to achieve the goal (Figs. 4, 5, 6, 7, 8, 9).

The first section of Table 1 clearly shows that there is a reduction in the considered fitness function, despite the fact that the values of the data traffic index and installation cost are not suitable.

The above mentioned values were also reduced when the PMU placement problem was added into WAMS, as seen in the second case of Table 1 (WAMS) by OARO. According to Table 1, for the 33-bus system with a control centre on bus 8, different locations are appropriate for deploying the PMUs. For PMUs at [2 5 6 7 9 12 15 18 21 24 27 30 33], $D_X = \sum S_X C_X^P$ where $X \in 2, 5, 6, 7, 9, 12, 15, 18, 21, 24, 27, 30, 33$; $i.e.\ D_X = S_2C_2^8 + S_5C_5^8 + S_6C_6^8 + S_7C_7^8 + S_9C_9^8 + S_{12}C_{12}^8 + S_{15}C_{15}^8 + S_{18}C_{18}^8 + S_{21}C_{21}^8 + S_{24}C_{24}^8 + S_{27}C_{27}^8 + S_{30}C_{30}^8 + S_{33}C_{33}^8$, which are briefly discussed on the first part of result analysis.

Previous research has shown in [47] that the addition of ZIB reduces the quantity of PMU required even further. As a result, with the addition of ZIB, the fitness function was reduced to ideal levels while also reducing the amount of PMUs. (as seen in the third portion of Table 1) represented in Fig. 3.

Also with the joint coordination of ZIB and WAMS further reduces the objective function. PMU locations for scenario 4 by OARO are [2, 5, 8, 11 14, 17, 21, 24, 26,

Table 1 Results for 33-bus test system

Random/Optimal solution		PMU placement PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBIps}}$)	Installation cost index($p.u.$)	Fitness value
PMU placement random solution		1 3 4 8 9 12 13 14 15 16 1 19 22 24 25 26 2 28 30 31 32	21	355	22.9000	21
		2 3 5 6 8 9 11 12 15 18 19 20 21 22 25 27 29 31 33	19	311	20.8000	19
PMU placement optimal solution	ARO	2 3 6 7 8 9 11 12 15 18 19 22 25 27 29 30 31 33	18	275	19.7000	18
	OARO	**1 2 3 4 7 10 12 15 18 20 22 23 24 27 30 33**	**16**	**268**	**17.4000**	**16**
Random/Optimal solution		WAMS PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBIps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 5 6 9 12 15 18 19 22 24 25 28 30 33	14	231	15.2000	288.2000
		2 5 6 8 9 12 15 18 19 22 24 28 30 33	14	218	15.3000	275.3000
PMU placement optimal solution	ARO	2 5 6 9 12 15 18 19 22 24 28 30 33	13	215	14.2000	268.2000
	OARO	**2 5 6 7 9 12 15 18 21 24 27 30 33**	**13**	**201**	**14.3000**	**254.3000**
Random/Optimal solution		ZIB PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBIps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 5 7 9 10 11 12 13 14 15 16 18 19 20 21 24 25 26 29 32	20	NA	NA	20
		1 2 4 5 6 7 8 9 10 11 14 17 21 23 25 28 29 32	18	NA	NA	18
PMU placement optimal solution	ARO	1 2 5 7 9 11 13 14 17 21 23 25 26 28 29 31 32	17	NA	NA	17
	OARO	**1 3 6 9 11 13 16 18 20 22 25 28 30 32**	**14**	**NA**	**NA**	**14**
Random/Optimal solution		WAMS with ZIB PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBIps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 5 8 10 12 15 18 21 22 24 27 29 30 33	14	234	15.2000	291.2000
		2 4 5 6 8 9 12 15 18 21 24 28 30 33	14	216	15.4000	273.4000
PMU placement optimal solution	ARO	2 5 8 912151821242730 33	12	193	13.1000	242.1000
	OARO	**2 5 8 11 14 17 21 24 26 29 32**	**11**	**192**	**12.2000**	**237.2000**

These are the optimal solutions of proposed methodology. So these are highlighted in bold

Fig. 4 69-bus distribution diagram

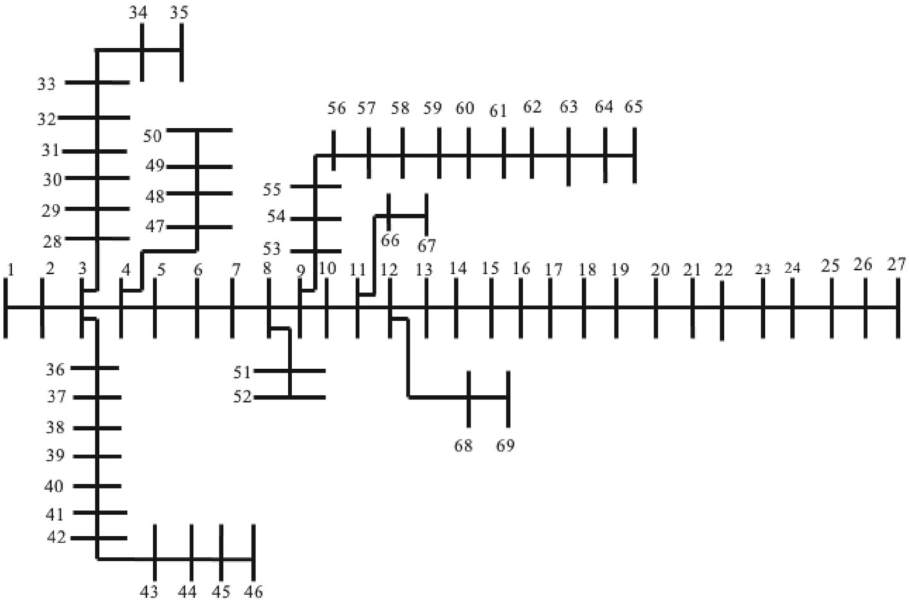
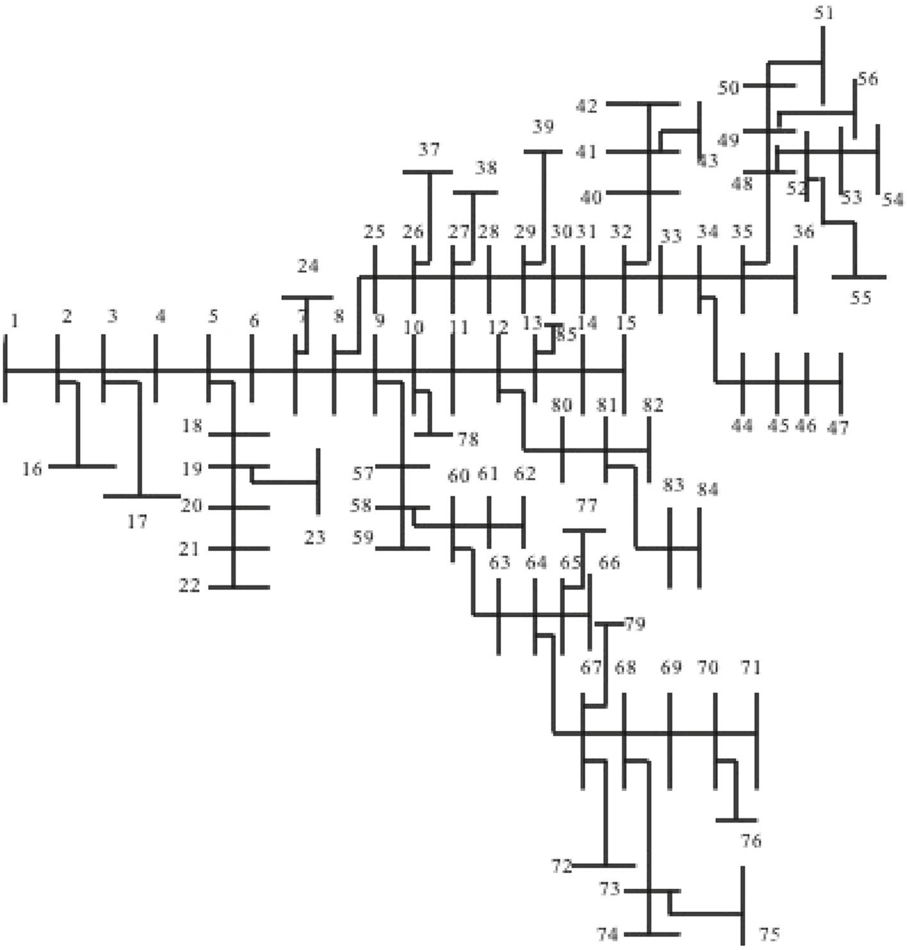


Fig. 5 85-Bus distribution diagram



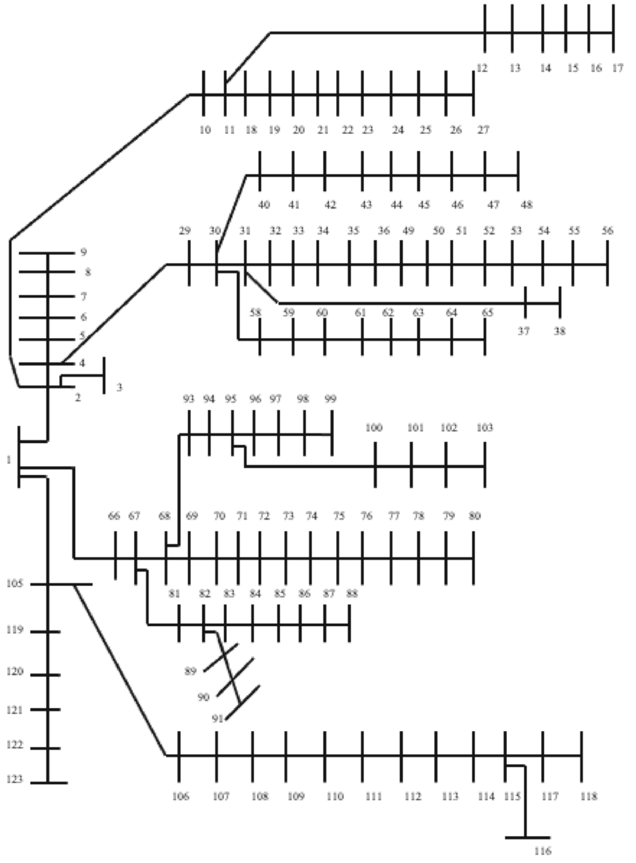


Fig. 6 118-Bus distribution diagram

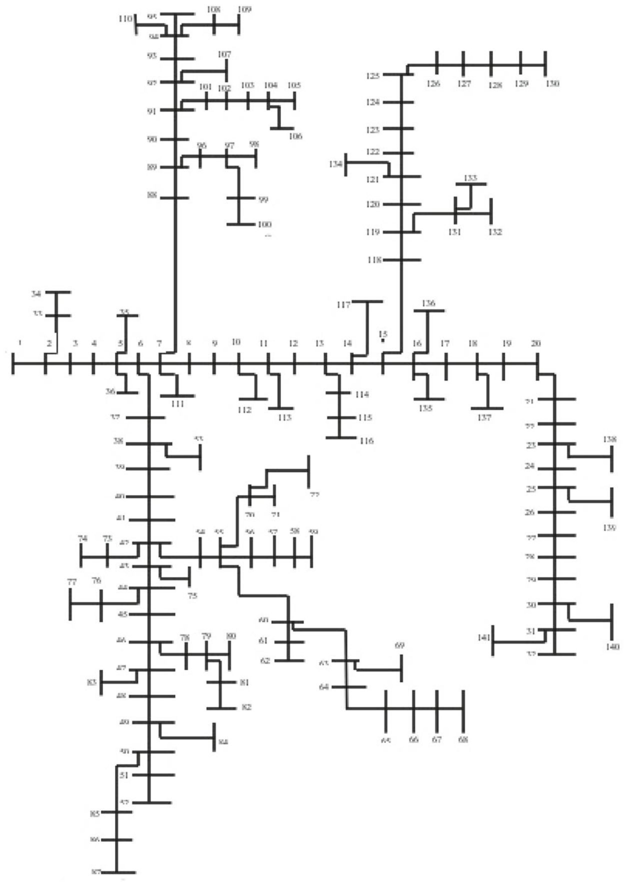


Fig. 7 141-Bus distribution diagram

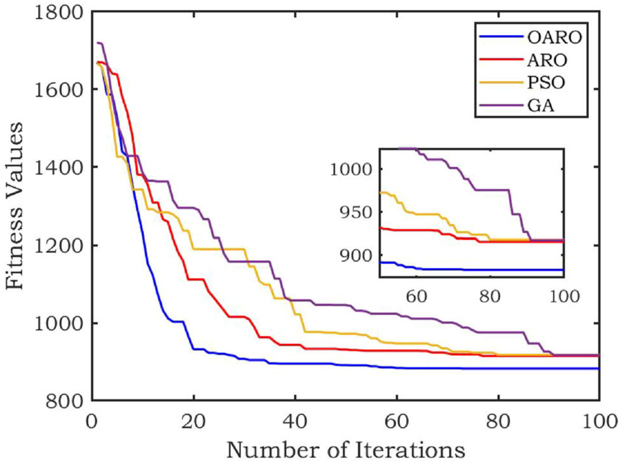


Fig. 8 Convergence graph of 85-bus case for WAMS

29, 32], where ZIB is present and WAMS is engaged. Therefore, the following details may be used. Now, the WAMS data traffic index can be obtained as: $D_X = \sum S_X C_X^P$ where $X \in 2, 5, 8, 11, 14, 17, 21, 24, 26, 29, 32$; $i.e.$ $D_X = S_2 C_2^8 + S_5 C_5^8 + S_8 C_8^8 + S_{11} C_{11}^8 + S_{14} C_{14}^8 + S_{17} C_{17}^8 + S_{21} C_{21}^8 + S_{24} C_{24}^8 + S_{26} C_{26}^8 + S_{29} C_{29}^8 + S_{32} C_{32}^8 = 192 PF_S kBlps$. According to Table 1, the overall installation cost index for scenario 4 is 12.2 $p.u.$ and fitness value is 237.2000. The same can be depicted in the last part of Table 1. It is clearly visible from there the incorporation of the proposed technique reduced the objective function to value in comparison with others. Table 1 has made it evident that for all four scenarios, OARO has performed better than alternative solutions. Figure 10 shows the implementation of ZIB and also the reduction of number of PMU is presented here. It is clear from the graphical depiction that, when ZIB (refer scenario 3 of Table 1 for numerical data) is added to the system, there are fewer PMUs than when ZIB was not added (refer scenario 1 of Table 1 for numerical data). In Table 2, the authors have depicted a optimal parameter selection scenario. Moreover, different permutation of input parameter selection for OARO have been shown. It can be clearly visible from the table the optimal fitness function has been obtained by considering the Population Size $= 50$, Iteration $= 100$, $J_r = 0.4$. Also in this scenario, the computational time has been obtained. The authors in the present work have used the same input parameters for the rest of succeeding research work to obtained the optimal solutions.

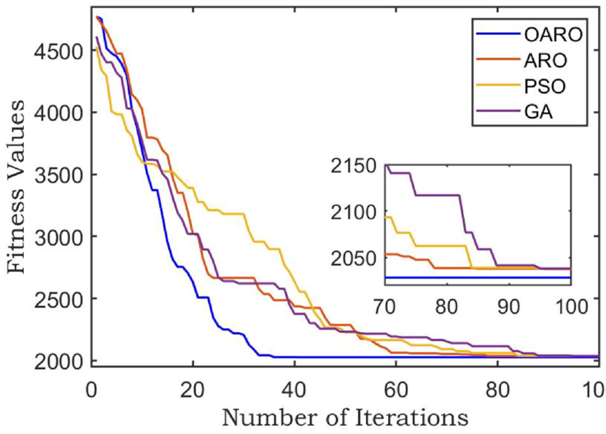


Fig. 9 Convergence graph of 141-bus case for WAMS in presence of ZIB

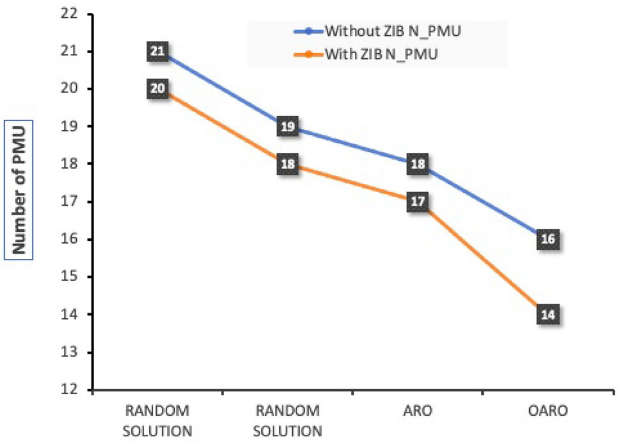


Fig. 10 Comparison of number of PMU, before and after use in ZIB for 33-bus

6.1.2 69-bus studied case

In this research, the 69-bus radial system (Fig. 4) is used to optimize three objective functions presented in Eqs. (1), (9) and (12) using an OARO algorithm to obtain the optimal PMU solutions that have minimum number of PMU, installation cost and fitness value. The data for network configuration of 69-bus test system along with the necessary assumption being made can be found in [48].

It is demonstrated in scenario 1 of Table 3 that the fitness value obtained by OARO, which is 26, is lower than that of other solutions.

In Sect. 2 of Table 3, the location of PMUs is displayed along with the data traffic index and installation cost index values. The indices, installation costs and fitness values of various combinations serve as boundaries. control centre, which is termed bus no 8. Just 26 places were determined to be the best ones for establishing the 69-bus systems. The PMUs can be deployed in a variety of places. When it comes to PMUs at [1 6 9 13 15 18 21 24 27 28 31 34 36 39 42 45 47 50 51 53 56 59 62 65 66 69], $D_X = \sum S_X C_X^P$ where $X \in$ 1, 6, 9, 13, 15, 18, 21, 24, 27, 28, 31, 34, 36, 39, 42, 45, 47, 50, 51, 53, 56, 59, 62, 65, 66, 69 *i.e.* $D_X = D_1 P_1^8 + D_6 P_6^8 + D_9 P_9^8 + D_{13} P_{13}^8 + D_{15} P_{15}^8 + D_{18} P_{18}^8 + D_{21} P_{21}^8 + D_{24} P_{24}^8 + D_{27} P_{27}^8 + D_{28} P_{28}^8 + D_{31} P_{31}^8 + D_{34} P_{34}^8 + D_{36} P_{36}^8 + D_{39} P_{39}^8 + D_{42} P_{42}^8 + D_{45} P_{45}^8 + D_{47} P_{47}^8 + D_{50} P_{50}^8 + D_{51} P_{51}^8 + D_{53} P_{53}^8 + D_{56} P_{56}^8 + D_{59} P_{59}^8 + D_{62} P_{62}^8 + D_{65} P_{65}^8 + D_{66} P_{66}^8 + D_{69} P_{69}^8$. The installation cost is 28.200 $p.u.$ as well. When compared to random solutions and solutions with ARO, this combination is deemed to be the best because it has the lowest installation cost index value and the least amount of data traffic with the proposed OARO technique. The graphical representation of no. of PMU, data traffic index, installation cost and fitness value are presented in Fig. 11. Here, the numbers of PMU, installation cost, data traffic index and fitness values have been shown along WAMS. The barchart depicts first two as the random solution followed by with ARO and finally with the proposed OARO technique. As can be visualize from the barchart that number of PMUs with the inclusion of WAMS through the proposed OARO technique came out to be 26 in comparison with ARO, i.e. 29 and other two random solution 32, 35, respectively. A similar type of response has been received by considering the other three scenario viz. installation cost, data traffic index and fitness values.

The quantity of PMU needed is further decreased to 25 number by OARO as well. As seen in the third section of Table 3, the addition of ZIB caused the fitness function to be reduced to optimal values while also reducing the number of PMUs.

The objective function is further diminished by ZIB and WAMS working together in concert. The final section of Table 3 illustrates the same. Figure 12 shows the combined implementation of WAMS and ZIB with a graphic depiction for case 4 of 69 buses. In this scenario also the proposed OARO yielded very promising solution. The adoption of the suggested technique lowered the objective function to value when compared to others, as is evident from there.

6.1.3 85-bus studied case

The proposed method is also used to the 85 bus distribution system to demonstrate its accuracy. [48] contains the network configuration data for the 85-bus test system as well as the requisite assumptions. Figure 5 displays a system diagram. When PMUs are placed at [2 3 5 9 12 14 21 23 24 26 29 32 36 38 42 43 44 46 49 51 53 55 58 60 62 65 67 70 73 76 78 82 83 85] the number of PMU acquired for Case 1 is 34. When PMUs placed at positions [2 3 7 9 10 15 16 18 21 23 26 27 30 33 36 39 41 44 47 49 51 54 55 59 62 63 65 67 70 73 80 82 84 85] along with a minimum installation cost of 37.4000 $p.u.$, the fitness values obtained for case 2 are 882.4000. The amount of PMU required is further reduced by

Table 2 Optimal parameter selection for OARO in 33-bus system with WAMS

Iteration	J_r Fitness value	Computation time (s)	Iteration	J_r Fitness value	Computation time (s)	Iteration	J_r Fitness value	Computation time (s)	Iteration	J_r Fitness value	Computation time (s)
Population = 25											
50	0.6263.87	12.32	50	0.5258.81	12.31	50	0.4257.89	11.31	50	0.3258.77	13.31
75	0.6262.77	13.33	75	0.5258.42	11.32	75	0.4255.03	10.33	75	0.3258.76	11.33
100	0.6261.98	11.38	100	0.5257.65	11.39	100	0.4255.04	12.38	100	0.3257.69	11.38
125	0.6261.07	12.41	125	0.5257.01	12.41	125	0.4254.98	11.41	125	0.3256.66	11.42
150	0.6259.84	11.43	150	0.5256.43	11.43	150	0.4254.87	12.43	150	0.3255.91	10.43
175	0.6259.87	11.49	175	0.5256.41	11.47	175	0.4254.79	11.49	175	0.3256.65	13.58
Population = 50											
50	0.6262.66	13.42	50	0.5259.43	13.42	50	0.4255.76	11.41	50	0.3256.76	12.41
75	0.6261.67	13.43	75	0.5259.67	13.44	75	0.4254.43	12.44	75	0.3256.13	11.16
100	0.6260.65	11.46	100	0.5258.74	12.45	**100**	**0.4254.3**	**10.13**	100	0.3255.54	13.46
125	0.6258.98	12.42	125	0.5258.31	11.41	125	0.4254.3	12.45	125	0.3254.89	12.45
150	0.6257.55	12.45	150	0.5257.56	14.46	150	0.4254.3	13.47	150	0.3254.89	11.46
175	0.6256.32	12.51	175	0.5255.45	12.5	175	0.4254.3	12.5	175	0.3254.87	13.52
Population = 75											
50	0.6263.87	14.52	50	0.5261.76	14.51	50	0.4256.89	15.51	50	0.3256.86	14.5
75	0.6262.89	13.53	75	0.5261.04	14.54	75	0.4254.99	13.53	75	0.3256.69	13.54
100	0.6261.67	13.56	100	0.5259.53	13.55	100	0.4254.88	12.56	100	0.3255.85	15.55
125	0.6260.88	13.58	125	0.5257.78	13.54	125	0.4254.87	13.52	125	0.3254.97	11.62
150	0.6259.43	134.61	150	0.5256.66	14.6	150	0.4254.65	14.59	150	0.3254.97	14.62
175	0.6258.45	14.62	175	0.5255.81	13.62	175	0.4254.87	13.63	175	0.3254.76	13.64

These are the optimal solutions of proposed methodology. So these are highlighted in bold

Table 3 Results for 69-bus test system

Random/Optimal solution		PMU placement PMU location	N_{PMU}	Data traffic index (PF_{SkBlps})	Installation cost index($p.u.$)	Fitness value
	PMU placement random solution	1 2 4 7 11 12 14 17 19 20 21 23 24 25 27 28 29 31 32 35 36 37 38 39 40 43 45 46 48 50 52 53 54 57 59 62 64 65 66 67 68 69	42	1018	45.6000	42
		1 5 7 10 14 17 20 23 26 27 29 30 31 34 36 39 41 43 45 47 49 51 54 57 58 60 62 64 65 66 67 68	32	773	34.8000	32
PMU placement optimal solution	ARO	1 5 7 9 13 16 18 21 24 27 29 31 34 36 38 41 44 46 48 49 50 51 55 57 60 62 65 66 67 69	30	669	32.4000	30
	OARO	**1 6 9 13 15 17 20 23 26 29 32 34 36 39 42 44 46 47 49 51 55 58 61 64 66 68**	**26**	**632**	**28.5000**	**26**
Random/Optimal solution		WAMS PMU location	N_{PMU}	Data traffic index (PF_{SkBlps})	Installation cost index ($p.u.$)	Fitness value
	PMU placement random solution	1 5 6 8 10 11 14 16 19 21 23 26 29 30 31 34 36 39 41 43 45 46 48 50 51 52 54 57 60 62 63 65 66 67 68	35	797	38.1000	940.1000
		1 4 7 8 11 14 17 19 21 22 23 24 26 27 29 32 35 37 39 42 45 49 51 52 54 56 59 62 63 65 67 69	32	796	34.8000	926.8000
PMU placement optimal solution	ARO	1 3 6 10 13 15 17 20 22 24 27 29 32 35 38 40 43 46 47 49 51 53 56 59 60 62 65 66 68	29	679	31.6000	797.6000
	OARO	**1 6 9 13 15 18 21 24 27 28 31 34 36 39 42 45 47 50 51 53 56 59 62 65 66 69**	**26**	**NA**	**NA**	**26**

Table 3 continued

Random/Optimal solution		ZIB PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
	PMU placement random solution	2 4 8 10 12 15 18 22 24 27 28 31 34 37 39 42 45 46 49 50 51 52 53 56 59 62 64 65 66 69	30	NA	NA	30
		1 4 8 10 12 15 18 19 22 23 24 27 28 31 34 37 39 42 45 46 50 51 54 56 59 62 65 66 69	29	NA	NA	29
PMU placement optimal solution	ARO	2 4 8 10 13 15 18 22 24 27 28 31 34 37 39 42 44 45 46 50 51 54 56 59 62 65 66 69	28	NA	NA	28
	OARO	**2 3 6 10 14 17 20 23 26 30 34 37 39 42 44 46 49 51 53 54 57 61 64 67 68**	**25**	**NA**	**NA**	**25**

Random/Optimal solution		WAMS with ZIB PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index($p.u.$)	Fitness value
	PMU placement random solution	2 4 8 10 12 15 18 22 24 27 28 31 34 37 39 42 45 46 49 50 51 52 53 56 59 62 64 65 66 69	30	NA	NA	30
		1 4 8 10 12 15 18 19 22 23 24 27 28 31 34 37 39 42 45 46 50 51 54 56 59 62 65 66 69	29	NA	NA	29
PMU placement optimal solution	ARO	2 4 8 10 13 15 18 22 24 27 28 31 34 37 39 42 44 45 46 50 51 54 56 59 62 65 66 69	28	NA	NA	28
	OARO	**2 3 6 10 14 17 20 23 26 30 34 37 39 42 44 46 49 51 53 54 57 61 64 67 68**	**25**	**NA**	**NA**	**25**

These are the optimal solutions of proposed methodology. So these are highlighted in bold

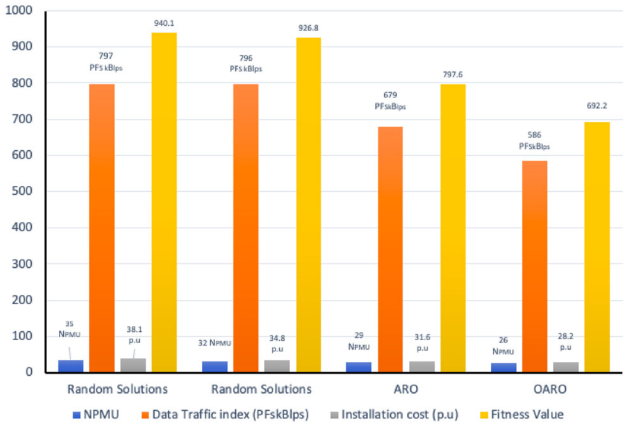


Fig. 11 69-Bus radial distribution system with WAMS

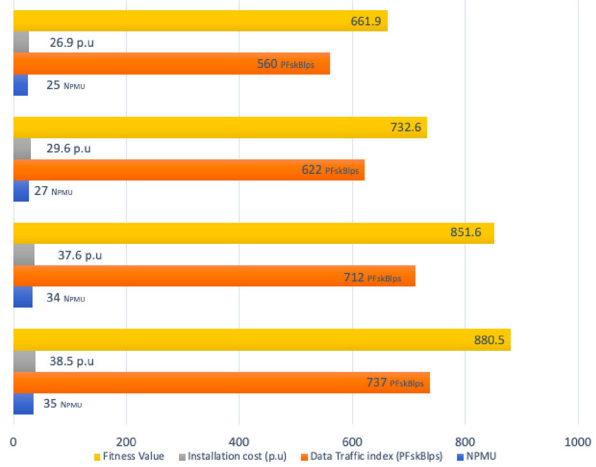


Fig. 12 Results of 69-Bus RDN with WAMS and ZIB

OARO to 26 numbers. The addition of ZIB led to the fitness function being reduced to ideal values while simultaneously reducing the number of PMUs, as shown in the third portion of Table 4. By combining the efforts of ZIB and WAMS, the goal function is further reduced to 731.9000 when PMUs are set at [2 3 6 9 12 14 19 22 25 28 30 32 35 37 42 46 48 51 54 58 62 66 67 70 74 82 84 85]. The same is demonstrated in the final portion of Table 4.

Figure 8 shows the convergence characteristics of 85-bus system considering the case of WAMS utilizing several optimization techniques like OARO, ARO, PSO and GA. It can be seen from Table 4 as well as Table 8 that the objective value or best fitness value obtained by OARO is 882.4000, whereas 915.1000 by ARO, 917.5102 and 917.0100 by particle swarm optimization (PSO) and genetic algorithm (GA), respectively. Also from the convergence graph depicted in Fig. 8, it is observed that the objective value by OARO converges systematically without any abrupt oscillations to the optimal solution than other optimization techniques. This validates the suggested approach convergence reliability. Therefore, based on the aforementioned simulation findings and convergence characteristics, it is possible to conclude that the suggested OARO approach is an effective method for placement of PMU in radial distribution system.

6.1.4 118-bus studied case

To show the suggested method's correctness, it is also applied to the 118-bus distribution system. Article [49] contained details about the system. There is a system diagram in Fig. 6. Control centre, which is termed bus no 856 PMU are acquired for case 1.

Only 47 locations of PMU are found to be ideal presented in case 2 of Table 5 for developing the 118-bus systems. The PMUs can be placed in various locations. $D_X = \sum S_X C_X^P \quad where \quad X = 2, 6, 8, 12, 15, 17, 18, 21, 24, 27, 28, 31, 33, 37, 39, 42, 45, 47, 50, 53, 54, 55, 57, 59, 62, 64, 67, 70, 72, 75, 77, 80, 82, 84, \quad 87, 89, 91, 92, 95, 98, 101, 104, 107, 110, 112, 114, 117$ with respect to PMUs at [2 6 8 12 15 17 18 21 24 27 28 31 33 37 39 42 45 47 50 53 54 55 57 59 62 64 67 70 72 75 77 80 82 84 87 89 91 92 95 98 101 104 107 110 112 114 117]. Additionally, there is a 51.4000 $p.u.$ installation fee. By using OARO, the required PMU is reduced to 47 digits, which is less than case 1.

As indicated in the third section of Table 5, the addition of ZIB resulted in the fitness function being reduced to ideal values and the quantity of PMUs being decreased.

The objective function is further minimised to 1658.4000 by combining the efforts of ZIB and WAMS when PMUs are set at [2 4 7 9 12 14 17 19 21 24 26 32 35 36 38 40 43 46 49 52 55 58 61 63 65 68 71 73 76 79 81 84 88 91 94 98 101 103 105 108 111 113 115 118]. The last section of Table 5 demonstrates the same. Figure 13 shows the integrated implementation of WAMS and ZIB with graphical representation for Scenario 4 of the 118-bus. It is clearly evident from Fig. 13 that the proposed OARO has yielded promising results in all the five scenarios by incorporating WAMS with ZIB. It is clear from the aforementioned four cases that the suggested OARO outperforms both alternative random solutions and ARO in terms of results.

6.1.5 141-bus studied case

To optimise the three objective functions described in Eqs. (1), (9) and (12), the OARO is finally applied to the 141 bus distribution system (Fig. 7) to produce the best PMU solutions with the least amount of PMU, the lowest installation cost, and the fitness value. The necessary presumptions and network setup data for the 141-bus test system are included in [49]. The fitness value obtained by OARO, which is 77, is shown in scenario 1 of Table 6 to be lower than that of other alternatives.

Table 4 Results for 85 bus test system

Random/Optimal solution		PMU placement PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index($p.u.$)	Fitness value
PMU placement random solution		1 2 3 4 5 6 7 8 10 11 12 13 14 15 16 18 20 21 22 23 24 25 26 29 30 32 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 63 64 65 66 68 69 70 72 73 74 75 77 78 79 80 81 82 83 84 85	74	1645	81.1000	74
		1 2 3 5 6 7 8 9 10 11 13 14 15 17 18 20 22 23 24 26 27 28 29 30 32 33 34 35 36 37 38 40 41 42 43 44 46 48 49 50 51 52 53 54 56 57 58 60 61 62 63 64 65 66 67 69 70 72 73 74 75 77 79 80 81 82 83 84 85	69	1541	76.2000	69
PMU placement optimal solution	ARO	1 2 4 6 8 10 15 16 17 18 19 21 22 23 24 26 28 29 32 35 38 41 44 46 48 49 51 53 54 55 57 59 60 62 65 66 70 72 73 74 77 79 80 82 84 85	46	994	50.0000	46
	OARO	**2 3 5 9 12 14 21 23 24 26 29 32 36 38 42 43 44 46 49 51 53 55 58 60 62 65 67 70 73 76 78 82 83 85**	**34**	**782**	**37.7000**	**34**
Random/Optimal solution		WAMS PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 3 6 9 10 11 14 15 18 20 22 23 24 25 29 32 36 37 38 42 43 44 47 51 52 54 56 59 61 63 66 68 71 72 73 76 77 78 79 81 83 85	42	816	45.0000	987.0000
		2 3 7 8 9 10 11 15 18 20 22 23 29 32 36 37 38 42 43 44 47 51 52 54 56 59 61 63 66 68 71 72 73 75 76 77 79 81 83 85	40	804	43.1000	967.1000
PMU placement optimal solution	ARO	1 3 4 7 10 14 16 19 22 26 29 32 35 38 41 44 46 51 54 55 56 57 59 60 62 65 69 70 72 74 75 79 80 82 84 85	36	768	39.1000	915.1000
	OARO	**2 3 7 9 10 15 16 18 21 23 26 27 30 33 36 39 41 44 47 49 51 54 55 59 62 63 65 67 70 73 80 82 84 85**	**34**	**743**	**37.4000**	**882.4000**

Table 4 continued

Random/Optimal solution		ZIB PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		1 4 5 6 7 9 10 12 13 15 17 19 21 23 24 26 27 28 29 30 34 35 37 40 41 42 44 47 48 50 51 53 58 59 61 62 63 65 66 67 70 71 72 73 74 76 79 81 82 84 85	51	NA	NA	51
		1 3 4 5 6 7 9 12 15 16 17 18 21 23 25 28 29 32 33 36 37 41 43 46 49 51 52 53 55 59 60 62 64 66 67 68 71 73 75 76 77 78 81 82 84	45	NA	NA	45
PMU placement optimal solution	ARO	1 5 8 11 15 17 19 21 26 29 31 33 35 41 46 50 53 58 61 65 69 71 72 74 82 84 85	27	NA	NA	27
	OARO	**2 3 6 10 14 21 23 25 27 30 32 35 37 42 46 50 53 59 62 65 67 70 74 80 82 83**	**26**	**NA**	**NA**	**26**
Random/Optimal solution		WAMS with ZIB PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 5 7 10 11 12 15 17 21 23 24 25 27 29 32 34 36 37 41 46 49 51 53 55 57 59 61 64 66 69 71 72 73 76 77 79 82 83 85	39	816	42.5000	975.5000
		2 5 7 8 10 11 15 17 19 22 29 32 35 37 38 42 43 44 47 51 54 55 56 58 62 63 65 68 71 72 74 75 76 79 81 83 85	37	759	40.000	910.0000
PMU placement optimal solution	ARO	1 3 7 9 12 15 16 18 19 22 25 28 30 33 35 37 38 39 41 45 47 51 54 55 56 59 62 63 65 67 70 74 75 78 82 83 85	37	754	39.9000	904.9000
	OARO	**2 3 6 9 12 14 19 22 25 28 30 32 35 37 42 46 48 51 54 58 62 66 67 70 74 82 84 85**	**28**	**617**	**30.9000**	**731.9000**

These are the optimal solutions of proposed methodology. So these are highlighted in bold

Table 5 Results for 118-bus test system

Random/Optimal solution		PMU placement PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index($p.u.$)	Fitness value
PMU placement random solution		1 2 3 5 7 8 9 11 12 13 14 15 16 17 18 19 20 21 22 25 26 27 29 32 34 35 36 37 39 41 42 43 45 46 47 48 50 52 53 55 56 58 60 61 64 65 67 68 69 70 71 72 73 76 78 79 80 81 83 84 85 87 91 92 94 96 99 100 101 103 104 105 106 107 108 110 111 112 113 114 116 117 118	83	2811	91.4000	83
		1 2 6 7 8 9 10 11 13 14 16 20 21 23 24 26 27 28 30 31 32 34 35 36 37 39 41 42 43 44 46 47 48 50 52 53 55 57 58 60 61 63 64 66 67 69 70 72 74 77 80 82 83 84 85 86 87 88 89 91 92 95 96 97 99 100 101 102 105 107 108 110 112 113 115 118	76	2519	83.4000	76
PMU placement optimal solution	ARO	1 2 4 6 7 9 12 13 15 16 18 20 22 24 26 29 31 33 37 40 42 44 45 46 47 50 52 53 54 57 59 61 62 63 65 66 68 71 73 75 77 79 80 83 85 87 88 91 94 96 99 101 102 103 106 107 110 112 113 114 117 118	62	2107	68.1000	62
	OARO	**1 2 4 6 9 12 15 16 18 20 22 23 26 29 31 33 37 40 43 44 45 47 50 52 53 54 57 59 61 65 66 68 70 71 72 73 75 77 79 80 83 85 88 91 94 96 99 101 103 106 107 110 112 114 116 118**	**56**	**1984**	**61.8000**	**56**

Random/Optimal solution	WAMS PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution	3 4 5 6 9 11 14 17 19 21 24 27 31 34 36 38 41 43 45 46 47 49 52 56 58 61 63 65 67 70 72 74 77 79 82 84 88 90 92 95 96 99 101 102 103 106 109 111 112 114 116 118	52	1691	56.6000	1903.5999
	22 6 8 12 15 17 18 21 24 27 28 31 33 36 37 39 42 44 45 47 50 53 54 55 57 59 62 64 67 70 72 75 77 80 82 84 87 89 91 92 95 98 101 104 105 107 110 112 114 117	50	1688	54.7000	1892.7000

Table 5 continued

Random/Optimal solution		WAMS PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index $(p.u.)$	Fitness value
PMU placement optimal solution	ARO	2 4 6 7 9 10 13 16 19 21 23 26 31 33 35 36 37 38 41 44 46 49 52 56 57 59 62 63 66 69 71 74 77 79 82 85 87 89 92 95 97 99 102 103 106 109 111 113 114 115 118	51	1607	55.5000	1815.5000
	OARO	**2 6 8 12 15 17 18 21 24 27 28 31 33 37 39 42 45 47 50 53 54 55 57 59 62 64 67 70 72 75 77 80 82 84 87 89 91 92 95 98 101 104 107 110 113 116**	**47**	**1984**	**61.8000**	**56**
Random/Optimal solution		ZIB PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index $(p.u.)$	Fitness value
PMU placement random solution		1 2 3 4 7 9 11 12 13 14 17 20 21 23 26 27 30 32 34 35 37 38 39 42 43 45 46 47 48 49 50 52 56 59 60 61 63 66 68 69 71 73 76 79 81 83 84 86 87 90 92 93 94 96 97 99 100 102 103 106 107 110 111 112 114 117	66	NA	NA	66
		1 2 4 7 9 11 12 13 14 17 20 21 23 26 27 30 32 34 35 37 38 39 42 43 45 46 47 48 49 50 52 56 57 59 60 61 63 66 68 71 73 76 79 81 83 84 86 87 90 92 93 94 96 97 99 102 103 106 107 110 111 112 114 117	64	NA	NA	64
PMU placement optimal solution	ARO	2 6 8 12 15 17 18 21 22 25 27 29 31 34 36 37 40 42 45 48 49 52 54 55 56 58 61 63 66 68 71 73 76 79 81 84 87 90 92 94 97 99 100 101 104 107 110 113 116	49	NA	NA	49

Table 5 continued

Random/Optimal solution	ZIB				
	PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
OARO	**2 6 8 12 15 17 18 21 22 25 27 29 31 34 36 40 42 45 48 49 52 55 58 61 63 66 68 71 73 76 78 79 81 84 87 90 92 94 97 99 100 101 104 107 110 113 116**	**47**	**NA**	**NA**	**47**

Random/Optimal solution		WAMS with ZIB				
		PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		2 6 8 12 15 17 18 21 24 27 28 31 33 37 39 42 45 47 50 53 54 55 57 59 61 62 64 67 70 72 75 77 80 82 84 87 89 91 92 95 98 101 104 107 110 112 114 117	48	1628	52.5000	1824.5000
		2 4 7 9 12 14 17 19 21 24 26 32 35 36 38 40 43 46 49 52 55 58 61 63 65 68 71 73 76 79 81 84 86 88 91 92 94 98 101 103 105 108 111 113 115 118	46	1550	50.600	1738.6000
PMU placement optimal solution	ARO	2 4 7 9 12 14 17 19 21 24 26 32 35 36 38 40 43 46 49 52 55 58 61 63 65 68 71 73 76 79 81 84 88 90 91 94 98 101 103 105 108 111 113 115 118	45	1511	49.5000	1695.5000
	OARO	**2 4 7 9 12 14 17 19 21 24 26 32 35 36 38 40 43 46 49 52 55 58 61 63 65 68 71 73 76 79 81 84 88 91 94 98 101 103 105 108 111 113 115 118**	**44**	**1478**	**48.4000**	**1658.4000**

These are the optimal solutions of proposed methodology. So these are highlighted in bold

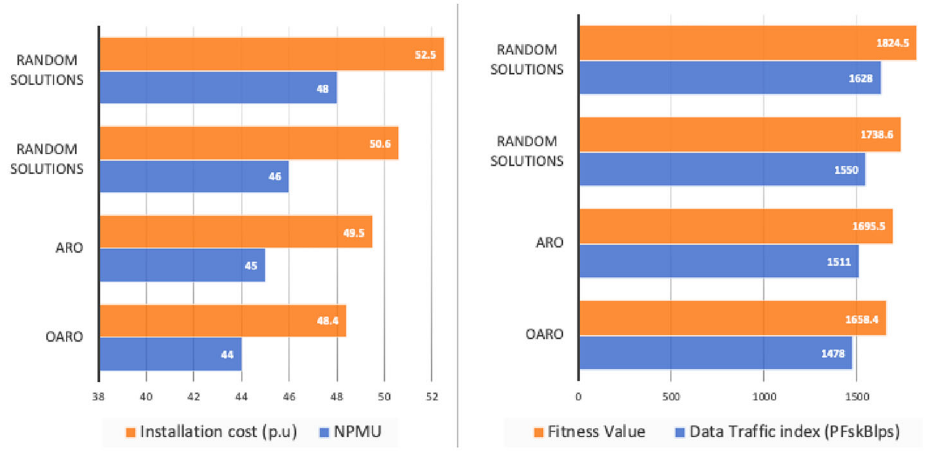


Fig. 13 Results of 118-bus RDN with WAMS and ZIB

The locations of PMUs are shown in Sect. 2 of Table 6, together with figures for the data traffic index and installation cost index. Boundaries are provided by the indices, installation costs, and fitness values of various combinations. control centre, often known as bus number 8. The PMUs can be placed in various locations. PMUs at [1 3 5 6 7 8 10 11 14 15 16 17 19 22 24 27 29 32 33 40 42 43 46 47 48 52 53 57 59 62 63 64 67 70 71 73 74 75 77 79 82 84 85 86 87 88 90 92 94 95 96 98 99 101 102 104 107 108 115 118 121 123 125 127 129 131 132 133 134 136 137 138 139 140 141] are concerned. $D_X = S_1C_1^8 + S_3C_3^8 + S_5C_5^8 + S_6C_6^8 + S_7C_7^8 + S_8C_8^8 + S_{10}C_{10}^8 + S_{11}C_{11}^8 + S_{14}C_{14}^8 + S_{15}C_{15}^8 + S_{16}C_{16}^8 + S_{17}C_{17}^8 + S_{19}C_{19}^8 + S_{22}C_{22}^8 + S_{24}C_{24}^8 + S_{27}C_{27}^8 + S_{29}C_{29}^8 + S_{32}C_{32}^8 + S_{33}C_{33}^8 + S_{40}C_{40}^8 + S_{42}C_{42}^8 + S_{43}C_{43}^8 + S_{46}C_{46}^8 + S_{47}C_{47}^8 + S_{48}C_{48}^8 + S_{52}C_{52}^8 + S_{53}C_{53}^8 + S_{57}C_{57}^8 + S_{59}C_{59}^8 + S_{62}C_{62}^8 + S_{63}C_{63}^8 + S_{64}C_{64}^8 + S_{67}C_{67}^8 + S_{70}C_{70}^8 + S_{71}C_{71}^8 + S_{73}C_{73}^8 + S_{74}C_{74}^8 + S_{75}C_{75}^8 + S_{77}C_{77}^8 + S_{79}C_{79}^8 + S_{82}C_{82}^8 + S_{84}C_{84}^8 + S_{85}C_{85}^8 + S_{86}C_{86}^8 + S_{87}C_{87}^8 + S_{88}C_{88}^8 + S_{90}C_{90}^8 + S_{92}C_{92}^8 + S_{94}C_{94}^8 + S_{95}C_{95}^8 + S_{96}C_{96}^8 + S_{98}C_{98}^8 + S_{99}C_{99}^8 + S_{101}C_{101}^8 + S_{102}C_{102}^8 + S_{104}C_{104}^8 + S_{107}C_{107}^8 + S_{108}C_{108}^8 + S_{115}C_{115}^8 + S_{118}C_{118}^8 + S_{123}C_{123}^8 + S_{125}C_{125}^8 + S_{127}C_{127}^8 + S_{129}C_{129}^8 + S_{131}C_{131}^8 + S_{132}C_{132}^8 + S_{133}C_{133}^8 + S_{134}C_{134}^8 + S_{136}C_{136}^8 + S_{137}C_{137}^8 + S_{138}C_{138}^8 + S_{139}C_{139}^8 + S_{140}C_{140}^8 + S_{141}C_{141}^8.$ The installation fee is 82.5000 $p.u.$ and fitness function is 2596.499 as well. When compared to random solutions and solutions with ARO, the proposed OARO is deemed to be the best because it has the lowest installation cost index value and the least amount of data traffic.

The quantity of PMU needed is further decreased to 55 number by OARO as well. As seen in the third section of Table 6, the addition of ZIB caused the fitness function to be reduced to optimal values while also reducing the number of PMUs. The objective function is further diminished by ZIB and WAMS working together in concert. The final section of Table 6 illustrates the same where fitness function is found 2028.3000.

The adoption of the suggested technique lowered the objective function to value when compared to others, as is evident from there.

The same is demonstrated in the final portion of Table 6. From the aforementioned four cases, it is obvious that the suggested OARO produces superior outcomes than alternative random solutions and ARO. Figure 9 shows the fitness function convergence characteristics of OARO, ARO, PSO and GA for considering WAMS with ZIB and the fitness function values are presented in Table 8. It can be seen from the graph, OARO convergence characteristic settles within 40 iterations, whereas ARO, PSO and GA settle after 60 and 70 iterations, respectively. The OARO algorithm's quick responsiveness is demonstrated by its convergent nature.

6.1.6 Performance characteristics and Statistical analysis

The computed results, as shown in Table 7, compare various PMU ideal sites with and without the use of the suggested OARO procedures. With the ideal PMU site, complete observability has been attained in addition to a decrease in WAMS data traffic and installation costs, which has led to a decrease in the fitness function. This raises the project's cost as a result. Table 7 makes it abundantly evident that the traffic generation that results from optimising the PMU count represents roughly one-third of the total traffic creation. Figure 14 depicts the comparing results of all busses after use of optimization technique (OARO). Moreover, in order to judge the superiority of the proposed method over PSO and GA, the said methods are successfully implemented on 33-bus, 69-bus, 85-bus, 118-bus and 141-bus and the statistical results with twenty five independent trials are illustrated in Table 8. The analysis has been carried on WAMS and WAMS along with ZIB separately.

In the first scenario, WAMS alone has been incorporated in the PMU placement problem. The authors have compared the results of the fitness function obtained by different well

Table 6 Results for 141-bus test system

Random/Optimal solution		PMU placement				
		PMU location	N_{PMU}	Data traffic index ($\mathrm{PF}_{\mathrm{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		1 3 4 5 6 7 9 10 11 17 18 20 21 22 24 25 26 29 30 31 32 33 34 35 36 38 39 40 41 43 44 45 47 49 50 52 54 57 59 60 61 62 63 65 67 71 72 74 75 76 77 78 79 81 82 83 84 85 87 88 89 92 93 94 96 98 99 100 101 104 105 106 107 109 110 111 112 113 114 115 116 117 118 121 122 124 125 126 127 130 132 133 135 136 138 139	96	2852	104.6000	96
		2 3 4 5 6 7 10 11 12 14 16 17 18 19 20 23 24 25 26 27 30 31 32 33 37 38 40 42 43 44 45 49 50 52 53 54 55 56 57 58 59 60 62 63 66 67 68 69 71 72 73 75 76 79 81 83 85 87 88 91 92 94 96 97 98 100 101 102 105 106 108 110 111 112 115 116 118 119 120 122 123 124 126 127 128 130 131 132 133 134 136 137 140 141	94	3000	104.1000	94
PMU placement optimal solution	ARO	1 2 3 5 6 9 10 11 12 14 16 18 19 21 24 25 27 28 30 31 32 34 35 38 39 41 42 43 44 45 47 48 49 51 53 54 56 57 59 62 63 64 66 68 70 71 72 73 74 76 78 80 81 83 85 86 88 91 94 95 97 100 102 104 105 107 108 110 111 113 114 115 117 118 119 120 121 122 125 126 127 128 129 131 133 138	86	2761	95.299	86
	OARO	**1 3 5 7 10 12 13 15 16 19 21 23 25 28 30 31 33 34 37 38 40 41 43 44 45 46 47 48 51 54 55 57 58 60 61 62 63 65 68 70 74 76 78 79 82 83 84 85 86 90 92 95 97 99 101 102 105 106 109 110 111 113 115 116 117 118 120 122 125 127 129 131 133 134 135 137 140**	**77**	**2458**	**85.2000**	**77**

Table 6 continued

Random/Optimal solution		WAMS				
		PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		1 3 4 5 6 7 8 9 12 13 16 17 18 20 23 25 27 29 30 32 33 34 35 37 38 40 41 43 44 47 48 49 51 54 57 58 61 63 66 67 70 71 74 75 77 79 81 83 84 86 87 88 89 90 91 93 95 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 116 117 118 120 123 124 125 127 130 131 132 133 134 136 137 140 141	91	4372	99.4000	2992.4000
		1 3 4 5 6 7 8 9 12 13 14 16 17 20 23 25 27 29 32 33 34 35 37 38 40 41 42 43 44 48 51 54 57 58 61 63 66 67 70 71 72 74 75 76 77 78 79 81 83 84 86 88 89 90 91 93 95 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 116 117 118 120 122 124 127 130 132 133 134 136 137 140 141	88	4372	95.8000	2753.8000
PMU placement optimal solution	ARO	1 3 5 6 7 8 10 11 14 15 16 17 19 22 24 27 29 32 33 40 42 43 46 47 48 52 53 57 59 62 63 64 67 70 71 73 74 75 77 79 82 83 84 85 86 88 90 92 94 95 96 98 99 101 104 106 107 108 112 115 118 121 123 125 126 127 129 131 132 133 134 135 137 138 139 140 141	77	4372	84.5000	2647.499
	OARO	**1 3 5 6 7 8 10 11 14 15 16 17 19 22 24 27 29 32 33 40 42 43 46 47 48 52 53 57 59 62 63 64 67 70 71 73 74 75 77 79 82 84 85 86 87 88 90 92 94 95 96 98 99 101 102 104 106 109 110 111 113 115 116 117 118 120 122 125 127 129 131 132 133 134 136 137 140**	**75**	**4372**	**82.5000**	**2596.499**

Table 6 continued

Random/Optimal solution		ZIB				
		PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		1 2 4 5 6 7 10 11 13 16 17 18 19 22 23 25 26 28 29 32 36 37 38 40 43 44 45 48 49 50 51 54 56 57 59 61 64 65 66 67 68 69 70 73 75 76 79 80 84 86 88 89 91 93 94 95 98 101 103 104 105 106 112 114 119 121 122 123 125 127 128 129 130 131 132 133 135 137 139	79	NA	NA	79
		1 2 4 5 6 7 10 13 16 17 19 22 23 25 26 28 29 31 36 37 38 39 40 43 45 48 50 51 54 57 59 61 62 65 66 68 70 73 75 76 77 79 84 86 87 88 89 91 92 93 94 98 101 103 105 112 121 123 125 128 129 130 131 132 133 135 137 139	68	NA	NA	68
PMU placement optimal solution	ARO	2 5 8 10 13 16 17 18 20 23 25 28 30 32 38 39 40 44 48 49 51 54 55 57 58 60 62 63 64 67 68 70 74 77 78 80 82 84 86 89 90 93 94 98 102 103 105 109 110 114 116 119 122 124 129 132 134 139	58	NA	NA	58
	OARO	**2 5 6 8 10 13 16 17 20 23 25 28 30 32 38 40 43 44 48 51 55 57 58 60 62 64 67 68 70 74 77 78 80 82 84 86 89 90 93 94 98 102 103 105 107 109 110 114 119 122 124 129 132 134 140**	**55**	**NA**	**NA**	**55**

Table 6 continued

Random/Optimal solution		WAMS with ZIB PMU location	N_{PMU}	Data traffic index ($\text{PF}_{\text{SkBlps}}$)	Installation cost index ($p.u.$)	Fitness value
PMU placement random solution		1 3 4 5 7 8 9 11 12 13 14 18 21 25 28 32 33 37 39 41 46 49 52 53 54 56 58 61 65 67 69 71 72 73 75 76 80 82 83 86 88 90 94 95 97 98 99 102 104 107 108 109 111 112 113 115 116 117 119 120 122 125 127 130 131 134 135 136 137 138 140 141	72	1932	78.1000	2226.0999
		1 4 7 8 11 12 16 18 19 22 23 27 30 32 34 35 36 37 38 40 43 46 49 52 54 56 59 62 63 66 68 70 73 77 80 81 83 85 87 89 94 98 99 101 104 107 109 112 114 115 117 118 119 122 125 128 130 132 133 134 139 141	62	1865	67.7000	2118.6999
PMU placement optimal solution	ARO	1 4 7 8 10 11 12 16 18 20 23 27 30 32 34 35 36 37 38 40 43 46 49 52 54 56 59 62 63 66 68 70 73 77 80 81 83 85 87 89 94 98 99 101 104 107 109 112 115 117 119 122 125 128 130 132 133 134 139 141	60	1792	65.6000	2037.6000
	OARO	**2 5 8 12 15 18 20 23 27 29 31 33 37 39 41 44 47 51 53 54 57 59 62 63 64 67 70 74 75 77 79 82 84 86 89 92 95 98 99 101 104 109 110 111 112 113 115 117 119 121 124 127 130 132 133 135 136 139 140**	**59**	**1787**	**64.3000**	**2028.3000**

These are the optimal solutions of proposed methodology. So these are highlighted in bold

Table 7 Performance comparison before and after the optimization of PMU

Test systems	Prior to PMU optimization				After PMU optimization			
	N_{PMU}	Data traffic index	Installation cost	Fitness value	N_{PMU}	Data traffic index	Installation cost	Fitness value
33-bus	33	541	36.1000	676.1000	11	192	12.2000	237.2000
69-bus	69	1642	75.7000	1924.69	25	560	26.9000	661.9000
85-bus	85	1857	93.3000	2205.30	28	617	30.9000	731.9000
118-bus	118	3968	129.599	4451.59	44	1478	48.4000	1658.4000
141-bus	141	4372	154.89	4949.89	59	1787	64.3000	2028.3000

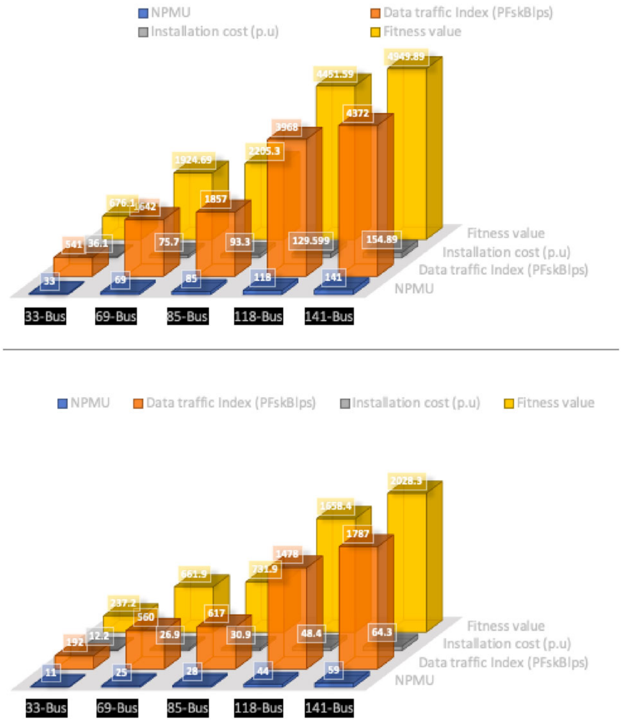


Fig. 14 Comparison of N_{PMU}, data traffic index, installation cost and fitness values of all busses

known optimization techniques. The benchmark of the best fitness function has been taken into consideration in which the difference between the best and worst value came out to be minimum. The population has been run for individual twenty five trials to obtain the best solution. Also the computational time obtained for each trails have been taken into account for justify the superiority of the proposed OARO technique. As it can be clearly depicted from Table 8 that the proposed OARO algorithm gain upper hand in satisfying all the considered criteria and reaching the optimal solution. The superiority has been same in all the test system considered.

In the second case (as illustrated in the lower section of Table 8), the PMU placement problem has been integrated with WAMS and ZIB. The fitness function results from several well-known optimization methods namely PSO, GA and ARO have been compared by the authors. The best fitness function benchmark, where the difference between the best and worst values is as little as possible, has been considered. In order to show the parity in the work, twenty-five individual trials have been conducted on the population in all the considered test systems in order to determine the optimal solution. The advantage of the suggested OARO technique has also been justified by taking into account the computational time achieved for each trail. Table 8 makes it abundantly evident that the suggested OARO algorithm triumphs in meeting every requirement and arriving at the best answer.

Table 8 Statistical analysis of five test systems using OARO, ARO, PSO and GA

Test systems	OARO Fitness value (WAMS)				ARO fitness value (WAMS)				PSO fitness value (WAMS)				GA fitness value (WAMS)			
	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)
33-bus	254.3000	254.4895	255.0781	10.13	268.2000	268.8999	269.4136	13.7	269.9472	271.4322	272.3039	21.43	269.8714	270.9788	271.9819	21.1
69-bus	692.2000	692.7998	693.0934	12.8	797.6000	798.4103	798.8799	17.3	798.9003	800.2777	801.4983	23.46	798.9100	800.5723	801.533	23.97
85-bus	882.4000	882.9031	883.3784	15.41	915.1000	916.2111	916.4946	19.1	917.5102	919.1109	920.4953	27.98	917.0100	918.4966	919.7455	26.7
118-bus	1781.4000	1781.9901	1782.5142	18.02	1815.5000	1816.4372	1817.1398	25.4	1817.9908	1819.6783	1821.2058	31.6	1817.4471	1819.1700	1820.5481	31.03
141-bus	2596.499	2597.2110	2597.729	21.97	2647.499	2648.5367	2649.4852	29.89	2650.1103	2651.9777	2653.4899	35.7	2650.5174	2652.4305	2653.9119	35.98
Test systems	OARO fitness value (WAMS with ZIB)				ARO fitness value (WAMS with ZIB)				PSO fitness value (WAMS with ZIB)				GA fitness value (WAMS with ZIB)			
	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)	Best	Average	Worst	CT (s)
33-bus	237.2000	237.6363	237.9925	12.2	242.1000	242.9106	243.5865	15.4	244.1345	245.7061	247.0699	23.21	244.1073	245.4908	246.8178	23.1
69-bus	661.9000	662.4301	662.8147	14.1	732.6000	733.5023	734.1625	19.12	735.1201	736.8230	738.2284	26.3	735.0997	736.7152	738.2304	26.5
85-bus	731.9000	732.5103	733.0695	17.5	904.9000	905.7688	906.7685	23.7	908.2153	909.9907	911.4979	31.5	908.2352	909.9867	911.4649	30.2
118-bus	1658.4000	1659.1100	1659.5932	19.9	1695.5000	1696.5163	1697.5326	27.83	1696.2101	1697.9921	1699.567	37.2	1696.2045	1697.7212	1699.435	34.9
141-bus	2028.3000	2028.9983	2029.5301	23.7	2037.6000	2038.844	2039.9984	34.29	2038.1301	2040.8578	2042.1653	39.5	2038.1131	2040.7996	2042.1559	39.9

The statistical results depicted in Table 8 clearly suggest the robustness and superiority of the suggested OARO method over PSO, GA and conventional ARO methods.

7 Conclusions and future research directions

In this article, oppositional ARO-based technique is used, for PMU placement on radial distribution system. Oppositional-ARO is a newly discovered optimization technique in the field of PMU placement for enhancing and stabilizing the voltage profile and to minimize the power loss of the system. The outcomes of the proposed OARO technique have been tested in five different test systems, namely 33-bus, 69-bus, 85-bus, 118-bus and 141-bus systems. Further, for different test systems, four separate scenarios have been considered. After determining the best location for the PMU, a wide area monitoring system (WAMS) is used to calculate the installation cost index, traffic congestion index, and related fitness function minimization. Thirdly, the use of zero injection bus (ZIB), which theoretically lowers the number of PMUs needed for the same problem, has expanded the scope of the current research. Lastly, the combined use of ZIB and WAMS in order to further reduce the goal function under consideration. This simulation results demonstrate how effectively the suggested strategy addresses the issue of optimized location PMU on distribution network. The outcomes of this research work are as under:.

1. The exploration characteristics of the suggested OARO help in identify the best location for PMU, which enhances exploring capacity and works to provide a worldwide solution.
2. It has been noted that the system's observability can be attained by incorporating WAMS.
3. By lowering the amount of PMU needed, the ZIB inclusion aids in obtaining the globally optimal solution.
4. The methods that are provided have an exploratory capability that helps handling the nonlinearity of large systems.
5. The proposed method has a high recurrence rate when applied to nonlinear problem solving.

Therefore, the conclusion may draw that the suggested method effectively manages the massive, intricate power system, which might persuade future researchers to apply the OARO algorithm in other domains. However, the following studies could be further investigated:

- It is possible to verify the flexibility and superiority of the suggested technique by accounting for different test systems.

- For real-time application, the suggested OARO algorithm might be improved.

In addition to above stated general discussion, some of the practical implementations of the PMU in radial distribution have also been stated below:

- Providing a dynamic picture of the system in both normal and abnormal operating modes, PMU aids in reducing imbalances brought on by the high penetration of distributed energy resources (DER) and enhancing system stability.
- PMU is utilized to handle the potential generation of phase angle deviation brought on by the addition of DER to the conventional radial distribution grid's overcurrent protection process.
- PMUs play a major role in state estimation because they make it possible to directly measure the state vector, which is composed of the power angles and magnitudes of the bus voltages. This greatly reduces the calculation time of the estimator while boosting its accuracy.

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Data availability The data that support the findings of this study are available on request from the corresponding author.

Declarations

Conflict of interest The authors attest that they do not own any specific communications or competing economic concerns that may be used to evaluate the accomplishment reported in the study work.

Ethical approval Several works involving mortal fields or animals realized by either author are not included in the study report.

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