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Reduced order modeling of delta operator systems by optimal frequency fitting approach

Arindam Mondal [a,*], Souvik Ganguli [b], Prasanta Sarkar [c]

[a] Dr. B C Roy Engineering College, Durgapur 713206, India
[b] Thapar Institute of Engineering and Technology, Patiala, Punjab 147004, India
[c] National Institute of Technical Teachers Training & Research, Kolkata, India

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ABSTRACT

The delta operator modeling provides a unified framework for both continuous-time and discrete-time modeling in system theory. At high sampling rate, the shift operator fails to provide meaningful information whereas, the delta operator parameterized system provides the same results as of continuous time systems. In this paper reduced order modeling of delta operator parameterized systems is considered. A complex domain (δ) optimal frequency matching (OFM) technique is proposed and frequency points are optimized using Particle Swarm Optimization (PSO) algorithm. This OFM is then utilized to find the reduced order model of the higher order system. PSO algorithm is a robust, global optimization technique, used to find these OFMs and thereby used to find the coefficients of the reduced order model by minimizing a cost function developed based on the responses of the higher order model and that of the reduced order model when both are excited by pseudo random binary sequences (PRBS). The performance characteristics are evaluated in software simulation using MATLAB considering example of higher order system in delta domain and time & frequency responses of the corresponding reduced model.

1. Introduction

Every physical system can be converted into mathematical model. Representation of the complex high order mathematical models impose lot of difficulty on simulation, analysis and control design. It is therefore requiring, finding the possibilities of some equation of the same type but of lower order and also they have the same dominant characteristics of the system under consideration. Reduced order modeling (ROM) is a systematic procedure to design and analysis of higher order systems. ROM finds its application in various fields of science and engineering like modeling of MEMS devices (Nayfeh, Younis, & Abdel-Rahman, 2005), chemical processes (Dorneanu, Bildea, & Grievink, 2009). Several researchers have developed different classical as well as nature inspired meta heuristic approaches for the last few decades to approximate high order system with the corresponding low order model (Fortuna, Nunnari, & Gallo, 1992; Jamshidi, 1983; Mahmoud & Singh, 1981) in both time and frequency domains (Biradar, Hote, & Saxena, 2016; Ganji, Mangipudi, & Manyala, 2017; Sikander & Prasad, 2017; Sikander & Thakur, 2018). Nowadays, artificial intelligence like deep learning based ROM using auto-encoders is also gaining attraction in the field of reduced order modeling (Halder, Fidkowski, & Maki, 2022).

Model order reduction problem in discrete time system has traditionally been studied using the shift operator and its associated transfer function description. With the inception of the delta operator in 1985, the system and control problems of discrete time systems are being looked into with renewed impetus using the delta operator formulation. The delta operator is essentially a finite difference operator, which has been widely used in numerical analysis. The advantages of the delta operator representation of the discrete time system over the conventional forward shift operator and its associated z-domain transformation is well documented in Goodwin and Middleton (1986, 1990, 1992), Neuman (1993a, 1993b). Mukhapadhyaya, in his work (Mukhopadhyay, Patra, & Rao, 1992) has shown that the so called delta operator is a particular case of the generalized δ- operator representation of the discrete-time systems. Due to superior properties of the delta operator systems along with its better finite word length effects (Maione, 2011), delta operator is a good choice among the researchers in the field of control theory and signal processing (Dolai, Mondal, & Sarkar, 2022; Mondal & Sarkar, 2016). The problems associated with traditional shift operator parameterization are thus can be avoided using delta operator

* Corresponding author.
E-mail addresses: arininstru@gmail.com, arindam.mondal@bcrec.ac.in (A. Mondal), souvik.ganguli@thapar.edu (S. Ganguli), sarkarprasant@gmail.com (P. Sarkar).

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parameterization and is used in numerous applications (Cortés-Romero, Luviano-Juárez, & Sira-Ramírez, 2013; Gao, Chai, Shuai, Zhang, & Cui, 2018; Lamrabet, Tissir, & Haoussi, 2020; Pal, 1996; Quezada-Téllez, Franco-Pérez, & Fernandez-Anaya, 2020; Sarkar & Pal, 2006; Zhao & Zhang, 2017).

An approximate frequency response (AFR) matching is a powerful tool, used in system and control theory. AFR matching method is used in this work to obtain the reduced order model in delta domain. The frequency response of the high order SISO system is computed in the delta domain at chosen frequency points. In this work, the optimal frequency points are fitted using Particle Swarm Optimization (PSO) algorithm.

The Particle Swarm Optimization (PSO) algorithm is a population based search algorithm formulated upon the simulation of social behavior of birds, bees or school of fishes. Each individual within the swarm represent by a vector in multidimensional search space. This vector has also one assigned vector which determines next movement of the particle called velocity vector. This algorithm also determines how to update velocity of particle. PSO was originally designed and introduced by Kennedy and Eberhart (1995, 2001) and find its applications in versatile area of science and technological research (Gad, 2022; Kondukwar & Dewangan, 2022; Roman, Marcin, & Robert, 2021; Yudong, Shuihua, & Genlin, 2015).

The following section discusses the significant contributions of this work. It can be observed from literature that the reduced order modeling (ROM) of the higher order systems have been done for continuous time systems and discrete time systems using the classical meta-heuristic approaches. This paper deals with model order reduction of the delta operator parameterized systems. In this work, the proposed reduced order model is obtained by optimal frequency fitting (OFF) in the delta domain using PSO algorithm. At very high sampling frequency, the discrete time higher order systems and corresponding reduced order systems are showing similar step and frequency response characteristics. Therefore, this method can be called as a unified method of model order reduction in delta domain using OFF and PSO algorithm. This method is new concept in the literature of reduced order modeling. An example is included to see how the time and frequency responses of the reduced order model are in close correspondence to that of the high order model.

The paper is organized as follows: fundamentals of delta operator are presented in Section 2. Section 3 describes model order reduction by approximate frequency response (AFR) matching method. In Section 4, Particle Swarm Optimization algorithm is discussed. Section 5 deals with optimal frequency fitting approach using PSO and Section 6 is devoted for result and analysis whereas conclusion is made in Section 7.

2. Fundamentals of delta operator

For the modeling of any continuous time dynamic system, $\frac{d}{dt}$ operator plays the pivotal role and it is defined as,

$$\frac{d}{dt} = \lim_{h\to 0} \frac{x_{(t+k)} - x_{(t)}}{h} \tag{1}$$

There is another operator which resembles the $\frac{d}{dt}$ operator functionally and structurally in the discrete domain known as delta-operator (δ) which is defined as,

$$\delta = \frac{q-1}{\Delta} \tag{2}$$

where, q is the forward shift operator; the sampling time is denoted by Δ.

It is an incremental difference operator that works on the principle of signal differentiator rather than traditional signal shifting operation as can be seen in case of shift operator parameterization. This is a shifted and scaled version of the shift operator. In the frequency domain, the delta operator is represented by γ and it is linearly related with the frequency domain variable (z) of discrete shift operator parameterization as given by (3),

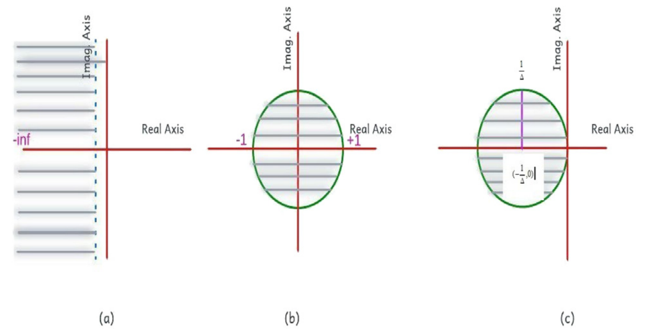

Fig. 1. (a) Stability zone: S domain, (b) Stability zone: Z domain, (c) Stability zone: δ-domain.

$$\gamma = \frac{z-1}{\Delta} \tag{3}$$

From (3), it can be observed that the stable zone in discrete delta domain lies in a circle of radius $1/\Delta$ and its center at ($-1/\Delta$, 0) corresponding to the stability region in z-domain. The stability regions in continuous time (s), discrete domain (z) and discrete delta domain (δ) are shown in Fig. 1a, b and c respectively. At very fast sampling rate ($\Delta \to 0$), the stability region of the delta operator parameterized system converges to the stability region of continuous time system as can be observed from Fig. 1.

The delta transform of any function g (kΔ) is defined as

$$G_\delta(\gamma) = \Delta \sum_{k=0}^{\infty} g(k\Delta)(1+\Delta\gamma)^{-k} \tag{4}$$

where, k is the indexing discrete-time parameter. Now if g (kΔ) is the impulse response of the system then $G_\delta(\gamma)$ is the delta transfer function of the system considering all the initial conditions are zero. Revisiting (3) and considering the relationship $z = e^{s\Delta}$, the frequency variable γ in discrete delta domain is represented by following equation.

$$\gamma = \frac{e^{jw\Delta} - 1}{\Delta} = |R_\delta|\, e^{j\theta} \tag{5}$$

Therefore, the complex delta domain transfer function can be represented by its magnitude and phase using (6).

$$G_\delta(\gamma) = |G_\delta| e^{j\varphi} \tag{6}$$

where, ω is the frequency in radian/sec of the input sinusoidal signal, $|R_\delta|$ and θ are the magnitude and phase of the transformed variable γ and $|G_\delta|$ and φ are magnitude and phase of the transfer function $G_\delta(\gamma)$. At fast sampling rate ($\Delta \to 0$), $\gamma = \frac{(e^{j\omega\Delta}-1)}{\Delta} \cong j\omega$ means the frequency response of the delta operator parameterized system converges to corresponding frequency response of the continuous-time system ($s = j\omega$).

3. Model order reduction using approximate frequency response (AFR) matching methods

The approximate frequency response (AFR) matching method in the δ domain is discussed in this section which will be used for the reduced order modeling of delta operator parameterized system. Let $G_\delta(\gamma)$ be the delta transfer function of the high order single input single output (SISO) stable discrete time system and given by (7).

$$G_\delta(\gamma) = \frac{G_{N\delta}(\gamma)}{G_{D\delta}(\gamma)} = K\frac{1 + b_1\gamma + b_2\gamma^2 + \cdots + b_m\gamma^m}{1 + a_1\gamma + a_2\gamma^2 + \cdots + a_n\gamma^n} \tag{7}$$

where, $m < n$ in consideration of a strictly proper system. It is assumed that $G_\delta(\gamma)$ is irreducible i.e. $G_{N\delta}(\gamma)$ and $G_{D\delta}(\gamma)$ have no zeros in common. Let $G_{R\delta}(\gamma)$ be the qth order reduced model of the form,

$$G_{R\delta}(\gamma) = \frac{G_{RN\delta}(\gamma)}{G_{RD\delta}(\gamma)} = K\frac{1 + \beta_1\gamma + \beta_2\gamma^2 + \cdots + \beta_p\gamma^p}{1 + \alpha_1\gamma + \alpha_2\gamma^2 + \cdots + \alpha_q\gamma^q} \tag{8}$$

where, $q \ll n$, $p \le q$ and for a strictly proper delta transfer function, $p = q - 1$. The order of the reduced model is assumed as q and this necessitates computation of at least $2q - 1$ free parameters of the reduced order model structure as shown by (8). For matching the frequency responses of $G_{R\delta}(\gamma)$ with $G_\delta(\gamma)$, let us consider Eq. (9).

$$G_\delta(\gamma)\big|_{\gamma=\frac{e^{j\omega\Delta}-1}{\Delta}} = G_{R\delta}(\gamma)\big|_{\gamma=\frac{e^{j\omega\Delta}-1}{\Delta}} \tag{9}$$

Using (5), (6) and (8), the following relationship is obtained.

$$\sum_{i=1}^{q-1}\beta_i|R_\delta|^i e^{j\theta i} - |G_\delta|e^{j\varphi}\sum_{i=1}^{q}\alpha_i\,|R_\delta|\,e^{j\theta i} = |G_\delta|e^{j\varphi} - 1 \tag{10}$$

Let us define $\psi = \omega\Delta$, therefore θ and φ become the function of ψ. Equating the real and the imaginary parts of (10), Eqs. (11) and (12) are obtained.

$$\sum_{i=1}^{q-1}\beta_i R_i(\psi) - \sum_{i=1}^{q}\alpha_i S_i(\psi) \cong T_i(\psi) \tag{11}$$

$$\sum_{i=1}^{q-1}\beta_i U_i(\psi) - \sum_{i=1}^{q-1}\alpha_i V_i(\psi) \cong W_i(\psi) \tag{12}$$

where, $R_i(\psi) = |R_\delta|^i\cos\theta i$, $S_i(\psi) = |G_\delta|\,|R_\delta|^i\cos(\theta i + \varphi)$,$U_i(\psi) = |R_\delta|^i\sin\theta i$, $T_i = |G_\delta|\cos\varphi - 1$, $W_i(\psi) = \sin\varphi$

The left-hand side(l.h.s.) expression of (11) and (12) are real function of ψ with unknown coefficients $\beta_{i\,\mathrm{and}}\alpha_i$. $T_i(\psi)$ and $W_i(\psi)$ are also two real (known) functions of ψ. Hence, the left hand side of (11) and (12) are designated as $\Phi_R(\psi)$ and $\Phi_I(\psi)$ respectively. Rewriting the Eqs. (11) and (12) for convenience as:

$$\Phi_R(\psi) = T_i(\psi) \tag{13}$$

$$\Phi_I(\psi) = W_i(\psi) \tag{14}$$

In order to force equivalence of two real functions $\Phi_R(\psi)$ and $\Phi_I(\psi)$ with their approximates $T_i(\psi)$ and $W_i(\psi)$ respectively, one may equate approximate number of initial few terms of the corresponding Taylor series expansions about $\psi = 0$. Thus, to accomplish appropriate matching of the left hand side functions in (13) and (14) with the corresponding functions on the right hand side, the initial N derivatives ($N \le q - 1$) of the corresponding functions are equated at $\psi = 0$ to formulate (15) and (16).

$$\frac{d^k}{d\psi^k}[\Phi_R(\psi)]\bigg|_{\psi=0} = \frac{d^k}{d\psi^k}[T(\psi)]\big|_{\psi=0} \tag{15}$$

$$\frac{d^k}{d\psi^k}[\Phi_I(\psi)]\bigg|_{\psi=0} = \frac{d^k}{d\psi^k}[W_i(\psi)]\big|_{\psi=0} \tag{16}$$

for $k \in [0, N - 1]$.$\Phi_R(\psi)$ approximately matches $T_i(\psi)$ if the following condition is satisfied.

$$\Phi_R(\psi)\big|_{\psi=\psi k} = T_i(\psi)\big|_{\psi=\psi k};\ k \in [0, N - 1] \tag{17}$$

where, ψ_k are small positive values around $\psi = 0$. Similarly, (18) is the condition to be satisfied for matching between $\Phi_I(\psi)$ and $W_i(\psi)$

$$\Phi_I(\psi)\big|_{\psi=\psi k} = W_i(\psi)\big|_{\psi=\psi k};\ k \in [0, N - 1] \tag{18}$$

The relations in (17) and (18) may be written in a matrix form as

$$Ax = b \tag{19}$$

It is clear from (19) that N values of ψ give $2N$ number of linear algebraic equations for the unknown parameters of the reduced model. For $(2q - 1)$ number of unknown parameters, N is at least equal to $(q - 1)$. In the case when, $2N > (2q - 1)$, the parameters of the reduced model may be determined by the least square solution of Eq. (19) as,

$x = (A^TA)^{-1}A^Tb$

where,

$$x = \begin{bmatrix}\beta_0 & \beta_1 & \beta_2 & . & . & . & \alpha_0 & \alpha_1 & \alpha_2 & . & . & . & . & . & \alpha_{q-1}\end{bmatrix}$$

$$b = \begin{bmatrix}T_0 & T_1 & T_2 & . & . & . & T_N & W_1 & W_2 & . & . & W_k & . & . & W_N\end{bmatrix}$$

4. Particle Swarm Optimization (PSO) algorithm

Particle Swarm Optimization (PSO) is a popular optimization tool for optimization of complex problems. This is a population based evolutionary algorithm. In PSO the particles are placed in search space of some problem function and each evaluates the objective function at its current location. Then each particle evaluates its movement through whole search space by its current and best locations, with some random value.

The PSO algorithm is simple and easy to implement. The procedures for implementing PSO (Kennedy & Eberhart, 1995) are as follows:

Step 1. Assume that, in d-dimensional search space and ith particle of the swarm can be represented by vector, $X_i = x_{i1}, x_{i2}, x_{i3}\,.........x_{id}$.

Step 2. The velocity of the particle is $V_i = v_{i1}, v_{i2}, v_{i3}........v_{id}$, where d is the dimension of the search space.

Step 3. For each particle, evaluate the fitness function $f(X_i)$ with d variable.

Step 4. Initialize the best visited position of the particle with $P_{i-best} = p_{i1}, p_{i2}, p_{i3}..........p_{id}$ and compare fitness evaluation with P_{i-best}.

If $f(X_i) < f(P_{i-best})$ then $f(P_{i-best}) = f(X_i)$, $P_{i-best} = X_i$

Step5. Initialize global best position $P_{g-best} = p_{g1}, p_{g2}, p_{g3}..........p_{gd}$. Identify the particle in the neighborhood with the best success so far.

If $f(X_i) < f(P_{g-best})$ then $f(P_{gi-best}) = f(X_i)$, $P_{g-best} = X_i$

Step 6. Position and velocity of the particle are updated by the following equation:

$$V_i(t+1) = w * V_i(t) + c_1 * R_1 * (P_{i-best} - X_i) + c_2 * R_2 * (P_{g-best} - X_i) \tag{20}$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \tag{21}$$

where, c_1, c_2 are positive constant. R_1, R_2 are two random variables with uniformly distribution. w is the inertia weight which shows the effects of previous velocity vector on the new vector. An upper bound is placed on velocity in all dimensions and is denoted by $V_{\max}$.

Step 7. Go to **step 3** until a criterion is matched, either a sufficiently good fitness or maximum no of iteration.

5. Optimal frequency fitting (OFF) using PSO algorithm

In this work, the reduced order modeling of higher order delta operator parameterized systems is proposed using optimal frequency fitting approach by PSO algorithm. The frequency points are chosen randomly in search space which is bounded by upper and lower value [1, 0]. The fitness values for every frequency points are evaluated using PSO. For the calculation of the fitness values, higher order delta parameterized system and model reduced order system are excited with the persistently exciting type pseudo random binary sequence (PRBS) to obtain the output. The PRBS input considered with a period of 'T'and bit interval equal to a scaled multiple of the sampling interval. The window size is considered as' W' as the fitness function is calculated for 'W' time steps of the input sequence. The algorithm for the proposed model order reduction using OFF method is described below:

Step 1: Initialize the PSO by setting the parameter of PSO (number of dimension, swarm size, maximum iteration, cognitive and social acceleration, initial and final value of iteration, maximum velocity).
Step 2: Initialize particle current position by selecting $(2q-1)$ arbitrary complex frequency points in the search space. The search space is bounded by upper and lower value of swarm.
Step 3: For each particle in the search space, compute its velocity.
Step 4: Find initial fitness value. To compute the fitness value following steps are followed:
(i) Discretize the higher order continuous time model incorporating sampler and hold with input and obtain the discrete-time model in the complex delta (γ) domain.
(ii) Assume the order of the reduced model and its structure as in (8).
(iii) Compute the non-zero entries, α_i, β_i by solving Eqs. (11) and (12).
(iv) Check the denominator polynomial of (8), $1+\sum_{i=1}^{q}\alpha_i\gamma^q$ for stability.
(v) Excite the systems by PRBS sequence and obtain the outputs.
(vi) Compute the performance index (fitness function) as given by (23)

$$PI = ER * ER^T \tag{22}$$

where, $ER = Y_\delta - Y_{R\delta}$.
Step 5: Compute the best particle in initial population by comparing the fitness value with best visited position so far. If fitness value is less than local best position value then update local best value and particle's position.
Step 6: Compare with global best position with fitness value. If fitness value is less than global best position then update global best value and global best position.
Step 7: Update velocity of the particle by (21).
Step 8: Update position of particle by (22).
Step 9: Go to **step 4** until best fitness value or maximum no of iteration is achieved.

6. Result analysis and discussion

To illustrate the method of reduced order modeling in δ domain and to observe the behavior of the resultant reduced order model an eighth order continuous-time system is considered (Sikander & Prasad, 2015) and described by (24). The corresponding delta operator parameterized transfer functions for different sampling time (Δ) are described by (25) given in Box I , (26) and (27).

The above systems are reduced to the corresponding 2nd order model using OFF and PSO. The algorithm parameters considered in this work is tabulated in Table 1.

Three delta operator parameterized systems are considered along with the original continuous time system for getting the corresponding reduced order model (2nd order) by proposed method. Different sampling times are considered to obtain the delta transfer functions. The value of sampling time (Δ) is reduced up to 0.001 s to unify the continuous time and discrete time results.

Table 1
PSO algorithm parameters.

Algorithm	Parameters	Values
PSO	Swarm size	30
	Maximum Velocity	100
	Maximum Iteration	100
	Acceleration constant ($c1$)	1.98
	Acceleration constant ($c2$)	1.98
	Initial inertia weight	0.96
	Final inertia weight	0.3

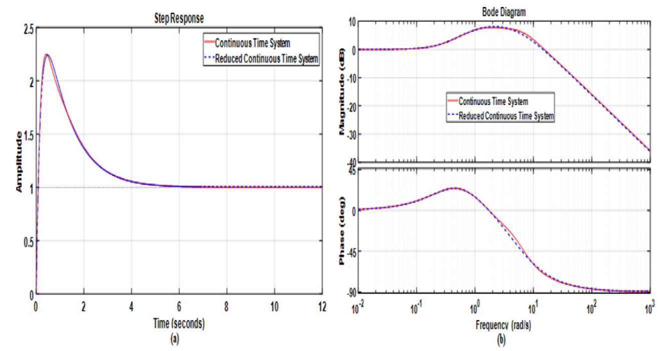

Fig. 2. (a) Step response of reduced order system and original system in continuous time domain (b) Frequency response of reduced order system and original system in continuous time domain.

Table 2
Statistical assessment of fitness function.

System	Best Value	Worst Value	Average Value	Standard Deviation
$G_{\delta 1}(\gamma)_{\vert\Delta=0.1}$	8.5218e−07	9.2317e−06	4.3319e−06	3.3586e−06
$G_{\delta 2}(\gamma)_{\vert\Delta=0.01}$	7.2381e−07	4.2381e−06	2.2381e−06	7.2381e−06
$G_{\delta 3}(\gamma)_{\vert\Delta=0.001}$	2.2453e−07	5.5435e−06	3.2381e−06	2.3424e−06

Table 3
p values for Wilcoxon signed rank test using PSO algorithm.

System	p- value
$G_{\delta 1}(\gamma)_{\vert\Delta=0.1}$	6.1835e−05
$G_{\delta 2}(\gamma)_{\vert\Delta=0.01}$	5.8343e−05
$G_{\delta 3}(\gamma)_{\vert\Delta=0.001}$	5.1843e−05

Pentium i7, 2.4 GHz processor with 32.0 GB RAM PC is used to perform the test of proposed algorithm using MATLAB R2020a version has bee.PSO is a stochastic metaheuristic method and it needs multiple run for getting statistical results. In this experimentation, 30 test runs are performed. The statistical results for the optimization of the fitness function are tabulated in Table 2 for the proposed algorithm. To validate the significance of the obtained results, non-parametric Wilcoxon signed rank test (Rosner, Glynn, & Lee, 2006) is considered and the p values are enlisted in Table 3.

The step response and frequency response of the original continuous time system and its reduced order model in continuous time domain using the proposed method are illustrated in Fig. 2a and b respectively.

The 2nd order reduced model of the three delta operator parameterized systems are tabulated in Table 4 along with the reduced order 2nd order model for original continuous tine systems using the proposed method of reduced order modeling.

$$G(s) = \frac{16s^7 + 483s^6 + 6010s^5 + 36380s^4 + 122700s^3 + 222100s^2 + 185800s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22450s^4 + 67280s^3 + 118100s^2 + 109600s + 40320} \quad (23)$$

$$G_{\delta 1}(\gamma)_{|\Delta=0.1} = \frac{12.145\gamma^7 + 279.4977\gamma^6 + 2604.3219\,\gamma^5 + 12588.137\gamma^4 + 34305.7376\gamma^3 + 51102.5552\gamma^2 + 36086.2484\gamma + 7253.946}{\gamma^8 + 27.6404\gamma^7 + 325.3555\,\gamma^6 + 2123.0418\gamma^5 + 8364.2883\gamma^4 + 20263.8888\gamma^3 + 29274.7067\gamma^2 + 22835.973\gamma + 7253.946} \quad (24)$$

$$G_{\delta 2}(\gamma)_{|\Delta=0.01} = \frac{15.5451\gamma^7 + 456.8906\gamma^6 + 5500.3765\,\gamma^5 + 32529.0874\gamma^4 + 107311.8647\gamma^3 + 190381.5754\gamma^2 + 156510.524\gamma + 33706.7324}{\gamma^8 + 35.0012\gamma^7 + 516.7752\,\gamma^6 + 4185.2051\gamma^5 + 20223.8615\,\gamma^4 + 59280.0208\gamma^3 + 101985.3025\gamma^2 + 92981.8399\gamma + 33706.7324} \quad (25)$$

$$G_{\delta 3}(\gamma)_{|\Delta=0.001} = \frac{15.9536\gamma^7 + 480.3233\gamma^6 + 5956.6639\,\gamma^5 + 35973.0938\gamma^4 + 121058.6417\gamma^3 + 218687.8729\gamma^2 + 182625.7406\gamma + 39601.0694}{\gamma^8 + 35.8982\gamma^7 + 542.9863\,\gamma^6 + 4499.4241\gamma^5 + 22215.4798\,\gamma^4 + 66429.1900\gamma^3 + 116371.5382\gamma^2 + 107804.2872\gamma + 39601.0694} \quad (26)$$

Box I.

Table 4
Reduced order systems in continuous time domain and discrete-delta domains with RMSE.

Sampling Time (Δ)	Reduced order System	Root Mean Square error
…..	$R(s) = \frac{15.4468\,s+4.9056}{s^2+6.168\,s+4.9056}$	2.16148e−02
0.1	$R_{\delta 1} = \frac{12.0605\gamma+3.6831}{\gamma^2+5.2206\,\gamma+3.6831}$	1.36858e−02
0.01	$R_{\delta 2} = \frac{16.5152\,\gamma+4.9251}{\gamma^2+6.7967\,\gamma+4.9251}$	1.1361e−02
0.001	$R_{\delta 3} = \frac{15.5883\,\gamma+4.9288}{\gamma^2+6.2543\,\gamma+4.9288}$	1.6952e−03

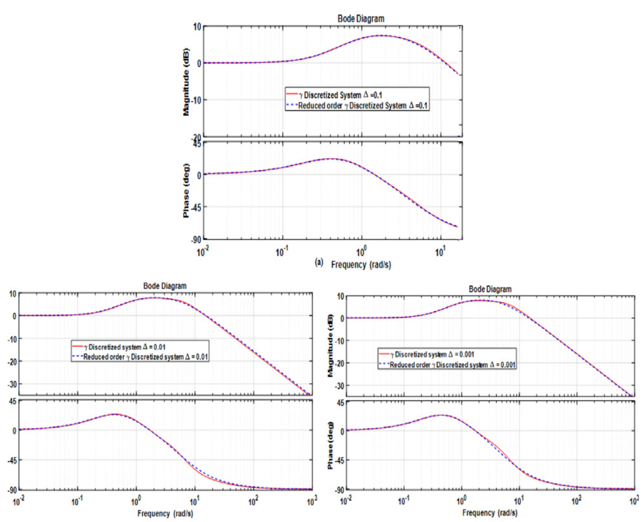

Fig. 3. (a) Frequency response of reduced order system $R_{\delta 1}$ in delta domain, $\Delta = 0.1$ (b) Frequency response of reduced order system $R_{\delta 2}$ in delta domain, $\Delta = 0.01$ (c) Frequency response of reduced order system $R_{\delta 3}$ in delta domain, $\Delta = 0.001$.

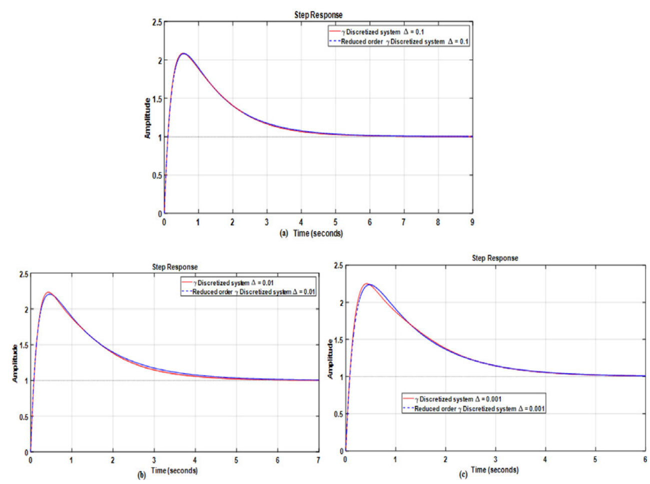

Fig. 4. (a) Step response of reduced order system $R_{\delta 1}$ in delta domain, $\Delta = 0.1$ (b) Step response of reduced order system $R_{\delta 2}$ in delta domain, $\Delta = 0.01$ (c) Step response of reduced order system $R_{\delta 3}$ in delta domain, $\Delta = 0.001$.

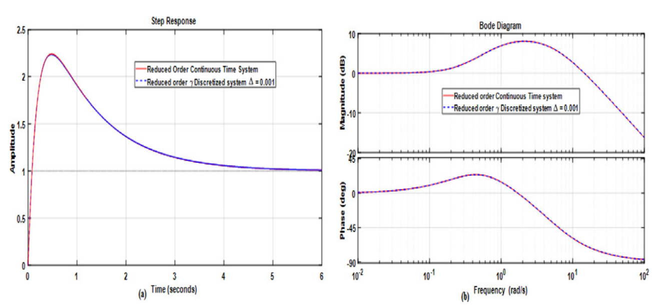

Fig. 5. (a) Step responses of reduced order systems in continuous time domain and in delta domain ($\Delta = 0.0001$) (b) Frequency responses of reduced order systems in continuous time domain and in delta domain ($\Delta = 0.0001$).

The frequency response analyses of the original as well as delta operator parameterized systems are illustrated in Fig. 3a, b, and c respectively for three different sampling times. All the three figures depict that the frequency response characteristics of reduced order models almost same to that of the original delta operator parameterized systems for low to high frequency range.

Fig. 4a, b and c are used to demonstrate the step responses of the reduced order models at different sampling instants. The step responses of the reduced-order delta operator systems are closely matching to its respective higher-order delta operator based systems as can be seen from Fig. 4a, b and c. It is revealed from Fig. 5a and b that the step response and frequency response in the reduced delta operator based system resembles to that of the continuous time reduced order model at fast sampling rate ($\Delta = 0.001$) making the method a unified one for reduced order modeling in delta domain using the proposed method.

In order to find the accuracy of results, Root mean squared error (RMSE) is evaluated in each case and tabulated in Table 4. RMSE is the square root of the mean of the square of all of the error. RMSE for each case have been calculated using Eq. (27).

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(P_i - R_i)^2} \quad (27)$$

Table 5
Comparison of time and frequency domain parameters for original and reduced order systems.

System	Rise time(s)	Settling time (s)	% Peak over-shoot	Gain margin (dB)	Phase margin (degree)
$G(s)$	0.0616	4.7856	125.1447	inf	109.5950
$R(s)$	0.0647	4.8637	124.2735	inf	110.2745
$G_{\delta 1}(\gamma)_{\mid \Delta=0.1}$	0.0830	5.1757	108.4863	inf	112.7936
$R_{\delta 1}$	0.0849	5.3789	108.2740	inf	113.0785
$G_{\delta 2}(\gamma)_{\mid \Delta=0.01}$	0.0635	4.8244	123.2712	inf	109.9299
$R_{\delta 2}$	0.0613	5.3857	122.6221	inf	110.1858
$G_{\delta 3}(\gamma)_{\mid \Delta=0.001}$	0.0618	4.7894	124.9553	inf	109.6288
$R_{\delta 3}$	0.0613	4.9857	123.6221	inf	111.1858

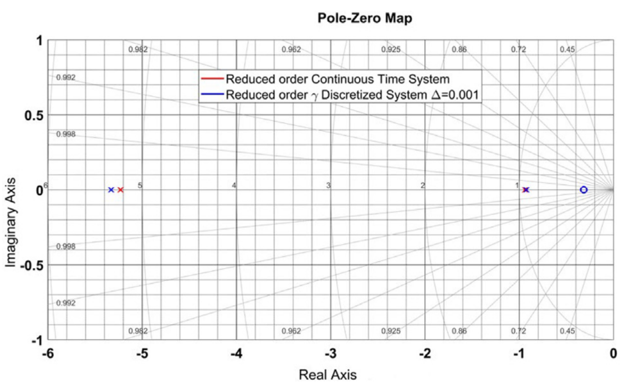

Fig. 6. Pole–zero plot of reduced order systems in continuous time domain and delta domain.

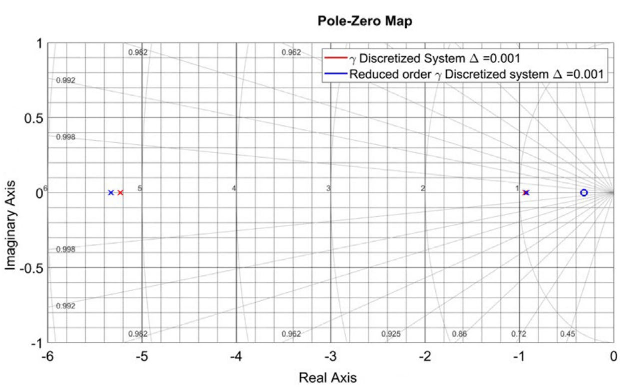

Fig. 7. Pole–zero plot of delta domain system and its reduced order model in delta domain.

where, N is the number of observations available for analysis, P_i is the vector of predicted values and R_i is the vector of the observed values.

In Table 5, the rise time, settling time, the maximum peak overshoot gain margin and phase margin of the original higher-order system and corresponding reduced order systems in both continuous time an delta domain are enumerated. From the obtained results as provided in Table 5, it has been found that the time domain and frequency domain parameters are very close for original and reduced order systems, thereby preserving the dynamic properties of original systems in the reduced order systems in delta domain as well.

The stability of the reduced order systems in delta domains are ensured from the pole–zero locations of the reduced order systems and corresponding pole–zero plots are illustrated in Figs. 6 and 7.

7. Conclusions

This work deals with the reduced order modeling of large scale SISO delta domain systems. The theoretical and simulation aspects of the reduced order modeling in the delta domain using optimal frequency fitting (OFF) approach with PSO algorithm are demonstrated. The reduced models are realized by optimizing the fitness values obtained through the optimal frequency fitting method using PSO. The PRBS responses are considered to study the responses. The step response and frequency response of the reduced order models in delta domain provides a replication of the same for original systems. From Table 5, it can be observed that the control parameters for original systems and corresponding reduced order models in delta domain are almost same. At higher sampling frequency the delta transformed reduced models are stable and nearly match the coefficients of the original-domain transfer functions. Wilcoxon signed rank test is performed and from Table 3, it is clearly concluded that the results obtained using the algorithm are significant for all the delta domain reduced order models. From Fig. 5, it has been observed that the step and frequency responses of the reduced order continuous time model and reduced order delta domain model at fast sampling time ($\Delta = 0.0001$ s) are matching closely. This leads to development of a unified method of model order reduction in delta domain. Therefore the frequency domain method presented in this paper is a viable alternative to other methods of model order reduction in the literature. This method can further be extended to design the reduced order model of fractional order, system with time delay and MIMO systems in future work.

CRediT authorship contribution statement

Arindam Mondal: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft. **Souvik Ganguli:** Conceptualization, Formal analysis, Investigation, Methodology, Validation, Visualization, Writing – review & editing. **Prasanta Sarkar:** Conceptualization, Methodology, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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