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## PROCEEDINGS

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# Optimal Biquadratic Approximation of the Fractional-Order Laplacian Operator Yielding Improved Constant-Phase Behavior

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***Abstract*—This paper deals with the biquadratic approximation of the fractional-order Laplacian operator $s^\alpha$, where, $\alpha \in (0, 1)$. The coefficients of the proposed rational function are optimally determined using an efficient constrained evolutionary algorithm. The proposed design constraint helps to attain the theoretical phase value of $\alpha\pi/2$ radians at the center frequency. Results showcase improved constant phase behavior for the proposed optimal approximants as compared to the models based on two different versions of the continued fraction expansion technique and the Sanathanan-Koerner curve-fitting method.**

***Index Terms*—constrained composite differential evolution, fractional Laplacian operator, fractional-order system, optimization, rational approximation**

## I. Introduction

The dynamical behaviour of a real-world system can be mathematically modeled using differential equations. Such equations are based on the traditional definitions of the differential and integral operators (also known as the differintegrals), which are the two fundamental operators of the integer-order calculus. The physical and geometrical interpretations of the differintegrals are also well-known to the scientific community. However, the generalization of classical calculus, where the order of the differintegral operator can be any number – integer or non-integer, real or complex, and rational or irrational, had its roots dated 300 years back. This branch of mathematics is known as the non-integer order calculus, or more popularly the fractional calculus [1]. Studies in various fields reveal that better representations of systems are obtained by describing their dynamics using the fractional-order (FO) differ-integrals, instead of the conventional integer-order ones [2].

Fractional derivative satisfies the following properties: (i) if the function is analytic, its derivative must also be analytic; (ii) for an integer type order, the FO derivative provides the same result as an ordinary derivative; (iii) a function's zeroth-order derivative results in no change; (iv) the fractional derivative operator is linear; and (v) for function $f(t)$, $D^\alpha D^\beta f(t) = D^{\alpha+\beta} f(t)$, for any $\alpha$ and $\beta$ [3].

Several definitions of the FO derivative exist in the fractional calculus literature. The Caputo definition, as given by (1), may be preferred since it employs integer-order initial conditions [4].

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{t_0}^{t} (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau \qquad (1)$$

where $f(t)$ is a causal function; $m-1 < \alpha \leq m$, and $m$ is an integer. If the initial conditions are zero, then the Laplace transform of (1) is $s^\alpha F(s)$.

There is no consensus among researchers regarding the physical interpretation of the fractional differintegrals, such as the area under the curve or slope of the tangent line at a point [5]. However, the importance of fractional calculus can be understood by considering a simple example, such as the FO differentiation of a periodic function $f(t) = \sin(t)$, as given by (2).

$$D_t^{\alpha} f(t) = \frac{d^\alpha}{dt^\alpha}\{\sin(t)\} = \sin(t + \frac{\alpha\pi}{2}) \qquad (2)$$

From (2), it can be inferred that the phase of the sine function varies proportionately with the order of differentiation. If $\alpha = 1$, then the classical relation $\frac{d}{dt}\sin(t) = \cos(t)$, is obtained.

A rational approximation of the FO Laplacian operator $s^\alpha$ helps transform a fractional-order transfer function (FOTF) into an integer-order system. A biquadratic transfer function can be represented as per (3).

$$H(s) = \frac{p_2 s^2 + p_1 s + p_0}{p_0 s^2 + p_1 s + p_2} \qquad (3)$$

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