AUSTRALIAN JOURNAL OF STRUCTURAL ENGINEERING
https://doi.org/10.1080/13287982.2023.2213506

ARTICLE

Stability of beam-column by geometrically nonlinear analysis

Sabyasachi Chandra

Department of Civil Engineering, Dr. B. C. Roy Engineering College, Durgapur, India

ABSTRACT

Stiffness properties of structural members, such as beam, plate and shell, can change drastically in the presence of axial forces due to geometric effects of the nonlinear strain components. In this paper, the stability behaviour of beam-column is investigated using the governing differential equation and compared with the geometrically nonlinear finite element analysis. The lateral deflection obtained from the theoretical model matches quite accurately with the numerical values for wide range of axial to critical load ratio P/P_{cr}. It is shown that bending stiffness decreases linearly with the axial load. By extending the theory, an expression for the membrane stiffness of the beam-column is presented in this paper. The geometrically nonlinear finite element analysis can capture exactly the parabolic variation of the membrane stiffness as per the derived expression. It increases initially up to $P/P_{cr} = 0.35$ and decreases rapidly to negligible value near the critical load indicating buckling instability.

ARTICLE HISTORY
Received 28 September 2022
Accepted 10 May 2023

KEYWORDS
Beam-column; bending stiffness; membrane stiffness; geometrically nonlinear analysis

1. Introduction

Impressive advances in the computational mechanics, material science and manufacturing processes allow structures to be designed closer to the limit of their load-carrying capacity. This leads to very slender and flexible structures whose behaviour becomes increasingly nonlinear with the applied load. The stability analysis of flexible structures is an interesting problem with practical implications.

In the classic work of Timoshenko and Gere (1961), the differential equations governing the bending deformation of beam-columns are presented and solved theoretically for few special cases. Barsoum and Gallagher (1970) formulated a finite element for non-uniform beams for torsional and torsional-flexural stability problems. As there could be many complex load combinations that may produce buckling in 3D beams, the behaviour of 3D beam-column is much more complex than 2D beam-column model. Chen and Atsuta (2008) derived the coupled differential equations that govern the behaviour of 3D beam-columns under biaxial bending and axial compression. Spillers and Rashidi (1997) described a computer program to generate the member stiffness matrix for geometrically nonlinear space frames using the equations of 3D beam-columns without cross-sectional warping. MacBain, Saadeghvaziri, and Spillers (1999) presented interaction curves that emphasised the phenomena of softening and hardening of the member stiffness due to initial prestress by solving the coupled differential equations using power series. Levy and Gal (2002) reformulated the basic four coupled differential equations governing the behaviour of 3D beam-columns to include varying cross-sections and solved them using the finite difference method.

Aristizabal-Ochoa (2004) used the method of elastica and elliptical functions in the large deflection stability analysis of slender beam-columns with rigid, semi-rigid and simple connections under combination of end loads. The drawback of elastica approach is that only flexural strains are considered and the effects of axial and shear strains are neglected. Therefore, the method gives closed-form exact solution for small to large curvature and transverse and longitudinal displacements for plane beam-columns under bending action only. Aristizabal-Ochoa (2008) proposed a new set of slope-deflection equations for Timoshenko beam-columns with semi-rigid connections by combining the effects of shear and bending deformations and including the effects of shear strains due to the applied axial force using Haringx's model. It is pointed out by Blaauwendraad (2008) that Haringx theory yields a wrong limit value for shear-weak beam-columns though it is reliable for helical springs. Based on the solutions of the governing differential equations for three different cases, it is suggested by Blaauwendraad (2010) that the stability of structural members should be investigated on the basis of Engesser theory and Haringx theory should be avoided.

Due to applied loads, all structures deform; however, deformations are generally small and their effect on the overall geometry of the structure can be neglected. Under such condition, the equilibrium

CONTACT Sabyasachi Chandra sabyasachi.chandra@bcrec.ac.in Department of Civil Engineering, Dr. B. C. Roy Engineering College, Durgapur 713206, India