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DESIGN AND IMPLEMENTATION OF FRACTIONAL-ORDER CONTROLLER IN DELTA DOMAIN

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Abstract. *In this work, a fractional-order controller (FOC) is designed in a discrete domain using delta operator parameterization. FOC gets rationally approximated using continued fraction expansion (CFE) in the delta domain. Whenever discretization of any continuous-time system takes place, the choice of sampling time becomes the most critical parameter to get most accurate results. Obtaining a higher sampling rate using conventional shift operator parameterization is not possible and delta operator parameterized discretize time system takes the advantages to circumvent the problem associated with the shift operator parameterization at a high sampling limit. In this work, a first-order plant with delay is considered to be controlled with FOC, and is implemented in discrete delta domain. The plant model is designed using MATLAB as well as in hardware. The fractional-order controller is tuned in the continuous domain and discretized in delta domain to make the discrete delta FOC. Continuous time fractional order operator ($s^{\pm\alpha}$) is directly discretized in delta domain to get the overall FOC in discrete domain. The designed controller in implemented using MATLABSimulink and dSPACE board such that dSPACEboard acts as the hardware implemented FOC. The step response characteristics of the closed-loop system using delta domain FOC resembles to that of the results obtained by continuous time controller. It proves that at a high sampling rate, the continuous-time result and discrete-time result are obtained hand to hand rather than the two individual cases. Therefore, the analysis and design of FOC parameterized with delta operator opens up a new area in the design and implementation of discrete FOC, which unifies both continuous and discrete-time results. The discrete model performance characteristics are evaluated in software simulation using MATLAB, and results are validated through the hardware implementation using dSPACE.*

Key words: *Continued fraction expansion, delta operator, dSPACE, fractional order controller*

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1. INTRODUCTION

A fractional-order system (FOS) is a system having a non-integer order differentiator and integrator. Nowadays FOS has become a vital research arena not only in mathematics but also in the system theory and control. From the literature, most of the real-world system is inevitably fractional order [1]–[3]. Since its inception in the year 1695, the mathematicians have done value addition and its utilization in control theory [4]. For the last few decades, the researchers have paid attention in modeling, analysis, simulation, solution of differential equations in fractional order domain to deliver a clear concept on FOS [5]–[7]. The control engineers are nowadays using the fractional-order calculus as a background of fractional-order controllers (FOC). To control the plant, the fractional-order controller becomes very much essential tools rather than the integer-order controller, and it is evident from the literature that the performance of the fractional-order controller is better than that of the integer-order controller [8]. The electrochemical process [9], dielectric polarization [10], visco-electric materials [11], chaos electromagnetic fractional poles [12], signal processing [13] are the primary areas where the fractional order calculus has been rigorously used for the last decade. In the case of FOS, the differentiator/integrator is symbolized by an irrational operator $s^{\pm\mu}$,where s is a complex quantity and known as Laplace transform variable. For the value of $\mu = \pm 1$ the irrational operator becomes an integer order operator $s^{\pm 1}$.The infinite dimensional irrational operator $s^{\pm\mu}$ is usually converted to the rational function either in a continuous domain (s-domain) or discrete domain (z or δ domain). To implement the fractional operators in the discrete domain, the discretization of the same operator is of primary concern [14]. The most common discretization method is Tustin operator-based discretization method. The comparative study between the different discretization methods in the z-domain is summarized in [14] to get the merits and demerits of each of the methods. For the realization of the fractional order operator in discrete domain, sampling rate during discretization should be at least 6-10 times the system bandwidth, as suggested by Shannon. But when the sampling rate is increased to a certain extent, corresponding z-domain transfer function becomes numerically ill conditioned thereby fails to provide meaningful insights.

The digital controller design in delta domain is better than the corresponding controller designed using shift operator [15]. The advantages of the delta operator parameterization are elaborated in [16], [17] particularly while the discrete z-domain results fails at high sampling rate. Delta operator has proven its potential for its application in control theory [18], system identification [19] in case of fault detection and network control [20], for Kalman filter-based controller design used in cyber-physical systems [21]. Direct discretization from continuous time domain to delta domain can make the procedure for FO controller design smoother and methods for the same has been proposed in [22], [23]. High speed digital realization for the fractional order operator can be possible using the properties of delta operator parameterization [24]. Moreover, delta operator parameterization has made it possible to understand both continuous and discrete-time systems in a unified framework.

For designing the fractional order controller, there are different works of literatures (AN231E04 Data sheet., 2012), [25]–[27]where different realization techniques are discussed. The tuning of parameters for the controllers is a fundamental issue. Several optimization techniques [28], [29] in the frequency domain [8], [30] are available. The analog realizations of fractional-order PID controllers have been proposed in [31]–[34].

Digital implementation of the FOC for Boost Converter using shift operator parameterization has been successfully done in [33]. Digital implementation of fractional-order controllers using FPGA via shift operator parameterization in indirect discretization domain is presented in [35], [36]. In this paper, DS1202 dSPACE board is a platform where a real-time controller in the discrete delta domain is implemented. In this paper, the performance of the proposed controller is studied using both simulations and digital hardware platforms, and a comparative study is done.

The significant contributions are made in this paper in manifold: In the earlier work, the fractional-order controllers are designed in different analog realization techniques. The discrete-time systems so far designed are done using shift operator parameterization, but shift operator parameterization fails to provide meaningful information at a high sampling rate. The real-time implementation of the controller in the digital domain needs a very high sampling rate to get a better result. In this work, the FO controller design for the integer-order plant with dead time is done using the delta operator parameterization and hardware realization is made using dSPACE. At a fast-sampling limit, the discrete domain results resemble to that of the continuous-time results providing a unified method of FOC design in delta domain. A new direct discretization method for discretizing the fractional order continuous time operator into discrete delta domain is utilized to obtain the rational transfer function in delta domain for the implementation using dSPACE board. Therefore, digital design and implementation of FOC using delta operator parameterization using dSPACE is a newer concept and a new direction for further research.

The organization of the paper is as: The basics of fractional-order system and controller are discussed in section 2. In Section 3, the discretization of fractional order operators using the delta operator is described. The digital realization of the FOPID controller using the delta operator is demonstrated in section 4. In Section 5, the implementation of the proposed controller in Simulink and dSPACE board is discussed. Finally, Section 6 & Section 7 is devoted to analyzing the result analysis and conclusion, respectively.

2. Fractional order system

2.1. Fractional order Calculus

In fractional calculus, the non-integer order differentiation/integration is denoted by a fundamental operator mD_{τ}^{ψ}, where ψ is used to specify the order of the operation like differentiation or integration. This operator is known as an integro-differentiator operator; this is mathematically represented as

$$mD_{\tau}^{\psi} = \begin{cases} \dfrac{d^{\psi}}{d\tau^{\psi}} & (\psi > 0) \\ 1 & (\psi = 0) \\ \int_{m}^{\tau} (d\tau)^{\psi} & (\psi < 0) \end{cases} \tag{1}$$

There are two popular definitions, such as Grünwald-Letnikov (GL) and Riemann-Liouville (RL) definitions, to express the integro-differentiator operator. (2) and(3) describe the GL and RL definitions, respectively.

GL definition:

$$mD_{\tau}^{\psi}\phi(t)=\lim_{p\to 0}p^{-\psi}\sum_{n=0}^{\left[\frac{\tau}{p}\right]}(-1)^{n}\binom{\psi}{n}\phi(\tau-np) \tag{2}$$

RL definition:

$$mD_{\tau}^{\psi}\phi(t)=\frac{1}{\Gamma(x-\psi)}\frac{d^{x}}{d\tau^{x}}\int_{m}^{\tau}\frac{\phi(p)}{(\tau-p)^{\psi-x+1}}dp \tag{3}$$

Where the value of ψ varies from $(x-1)$ to x and Γ is used to represent the Euler's gamma function.

2.2. Fractional order differential equation and transfer function

The fractional-order differential equation is used to describe the dynamics of a fractional-order system (FOS). Likewise, with the case of the classical integer order system, the Laplace transform of the fractional-order differential equation generates the transfer function of the FOS. The mathematical equation of a fractional-order system is described by (4).

$$\begin{aligned}&a_{n}D^{\psi_{n}}y(t)+a_{n-1}D^{\psi_{n-1}}y(t)+\ldots\ldots+a_{0}D^{\psi_{0}}y(t)=b_{m}D^{r_{m}}u(t)\\&+b_{m-1}D^{r_{m-1}}u(t)+\ldots\ldots+b_{0}D^{r_{0}}u(t)\end{aligned} \tag{4}$$

Where, $D^{\psi}\equiv{}_{0}D_{t}^{\psi}$ is known as RL-derivative or Caputo fractional derivative. The input and the output of the system are denoted by $u(t)$ and $y(t)$ respectively, $a_i(i = 0,......,n)$ and $b_i(i = 0,......,m)$ are constants and $\psi_i(i = 0,......,n)$, $r_i(i = 0,......,m)$ are arbitrary real numbers. In general, the values of ψ_i and r_j can be considered as $\psi_n > \psi_{n-1} > ..... > \psi_0$, and $r_m > r_{m-1} > ..... > r_0$. Laplace transform of (1) gives rise to a continuous-time transfer function as given by (5).

$$L\{mD_{\tau}^{\psi}\phi(t)\}=S^{\psi}\phi(s) \tag{5}$$

According to the definition of Caputo, the fractional derivative m is taken equal to 0, and the Laplace transform $\phi(t)$ is denoted by $\phi(s)$.

By using the expression as derived in (6), Laplace transform is applied on both sides of the (4) gives rise to the transfer function of a system with $y(t)$ as the output and $u(t)$ is the input.

$$G_{0}(s)=\frac{Y(s)}{U(s)}=\frac{b_{m}s^{r_{m}}+b_{m-1}s^{r_{m-1}}+\ldots\ldots+b_{0}s^{r_{0}}}{a_{n}s^{\psi_{n}}+a_{n-1}s^{\psi_{n-1}}+\ldots\ldots+a_{0}s^{\psi_{0}}} \tag{6}$$

where, $U(s) = Lu(t)$ and $Y(s) = Ly(t)$,

2.3. Fractional Order PID controller ($PI^{\lambda}D^{\mu}$)

The fractional order $PI^{\lambda}D^{\mu}$ controller performs better than the integer-order PID controller owing to its greater number of degrees of freedom. In case of the FOPID controller, the orders of the Integrator and Differentiator $(\lambda < 0,\ \mu > 0)$ are non-integer.

Therefore, by using the fractional-order calculus for differentiation, integration and Laplace transform, the continuous-time domain transfer function of fractional order $PI^{\lambda}D^{\mu}$ controller gets the following form:

$$G_c(s) = \frac{U(s)}{E(s)} = K_p + K_i s^{-\lambda} + K_d s^{\mu} \ (\lambda, \mu > 0) \tag{7}$$

where, $U(s) = Lu(t)$ and $E(s) = Le(t)$ are output and the input of the controller, respectively.

The integer-order PID controller can be obtained by using $\lambda = 1$ and $\mu = 1$ in (7). Likewise, the PD controller can be obtained if the value of $\mu = 0$, and $K_i = 0$. This may conclude that (7) is the generalized transfer function of integer/fractional-order controller. The basic structure of the FOPID controller is given in Fig. 1.

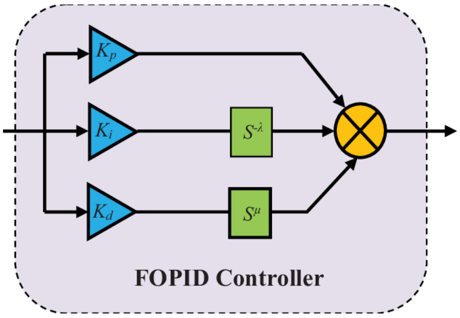


Fig. 1 Fractional order $PI^{\lambda}D^{\mu}$ Controller

3. Direct Discretization of Fractional Order Integrator and Differentiator Using Delta Operator

3.1. Relationship between s-domain and γ -domain

The shift operator parameterization is used to describe the discrete-time system. The forward shift operator is usually denoted by q. The delta domain is an area where discrete-time systems are represented using the delta operator δ. The delta operator (δ) is nothing but the scaled and shifted version of the forward shift operator (q). The δ-operator is related with the forward shift operator q as (Δ is the sampling time).

$$\delta = \frac{q-1}{\Delta} \tag{8}$$

At high sampling period ($\Delta \to 0$), the following identity is obtained when delta operator is applied on a differentiable signal $y(t)$:

$$\lim_{\Delta \to 0} \delta\, y(t) = \frac{y(t+\Delta) - y(t)}{\Delta} = \frac{d}{dt} y(t) \tag{9}$$

The continuous-time derivative can be obtained from the delta operated signal at a fast-sampling limit as can be seen from (9). The relationship between the frequency variable $'\gamma'$ in the delta domain and the frequency variable $'z'$ of the shift operator domain is given below:

$$\gamma = \frac{z-1}{\Delta} \tag{10}$$

In (10), replacing $z = e^{s\Delta}$, the relationship between the frequency variables in continuous time and discrete delta time is obtained and is depicted by (11).

$$\gamma = \frac{e^{s\Delta}-1}{\Delta} \ ,$$

or,

$$e^{s\Delta} = 1+\gamma\Delta$$

$$s = \frac{1}{\Delta}\ln(1+\gamma\Delta) \tag{11}$$

Equation (11) represents the direct relationship between the variable s and γ.

3.2. Direct discretization of fractional order operator in delta domain

For the realization of FOC in delta domain, discretization of the fractional order operator ($s^{\pm\mu}$) in delta domain plays the pivotal role. From (11), the transformation of the fractional order operator into delta domain from continuous time domain can be re-established as:

$$s^{\pm\mu} = \left(\frac{1}{\Delta}\ln(1+\gamma\Delta)\right)^{\pm\mu} \tag{12}$$

By using trapezoidal quadrature rule [37] and CFE, ln (1 + x) function can be successfully approximated to its closed form is as follow:

$$\ln(1+x) = \frac{6x+3x^2}{6+6x+x^2} \tag{13}$$

Replacing x by $\gamma\Delta$ in (13), (11) can be rewritten as

$$s = \left\{\frac{1}{\Delta}\ln(1+\gamma\Delta)\right\} \approx \left\{\frac{6\gamma+3\Delta\gamma^2}{6+6\gamma\Delta+\Delta^2\gamma^2}\right\} \tag{14}$$

From (14), it is evident that at fast sampling rate ($\Delta \to 0$), $s \approx \gamma$ meaning, the continuous and discrete delta domain becomes replicate to each other, thereby (14) gives a direct relationship between the two domains.

Equation (12) can be rewritten as:

$$s^{\pm\mu} = \left\{\frac{6\gamma+3\Delta\gamma^2}{6+6\gamma\Delta+\Delta^2\gamma^2}\right\}^{\pm\mu} \tag{15}$$

Rational transfer function in delta domain corresponding to any fractional order operator can be realized using (15) through the direct discretization method as demonstrated in [23]

In continuous-time system representation, fractional-order differentiator (FOD) and fractional-order integrator (FOI)are mathematically expressed as:

$$G_d(s) = s^r (0 < r < 1) \tag{16}$$

$$G_i(s) = s^{-r} (0 < r < 1) \tag{17}$$

Continued Fraction Expansion (CFE) [38], [39] is used as a powerful tool that operates on the generating function to get a rational transfer function.

The CFE approximation is mathematically formulated using (18)[39].

$$(1+p)^q = 1+\cfrac{qp}{1+\cfrac{(1-q)p}{2+\cfrac{(1+q)p}{3+\cfrac{(2-q)p}{2+\cfrac{(2+q)p}{5+\cfrac{(3-q)p}{2+.....}}}}}} \quad (18)$$

To obtain the standard form of CFE as given in (18), p is replaced by $\left\{\left(\frac{6\gamma+3\Delta\gamma^2}{6+6\gamma\Delta+\Delta^2\gamma^2}\right)-1\right\}$ to get the result obtained by CFE in (15).

Here, (15) is used as the generating function for the integer order approximation of the fractional-order differentiator/integrator in the delta domain as mathematically represented by (19).

$$G_{del}(\gamma) = CFE\left(\frac{6\gamma+3\Delta\gamma^2}{6+6\gamma\Delta+\Delta^2\gamma^2}\right)^{\pm r} \quad (19)$$

In this work, third order approximation of FOD and FOI are considered for the realization and implementation purpose. Delta domain coefficients [23] for the third order approximation of s^r are tabulated in Table 1.

Table 1 Delta-Domain coefficients for third-order approximation of s^r

$Dnum_3 = (3/\Delta)r/(r+1)/(4096r^6 + 26624r^5 + 9472r^4 - 201472r^3 - 252944r^2 + 331304r + 506955)$

Coefficient	Numerator
H_0	$(30720r^6 + 454416r^3 - 36096r^5 - 838259r + 78360r^2 - 4096r^7 - 192000r^4 + 506955)Dnum_3$
H_1	$(-938460\Delta r + 1388142\Delta - 723408\Delta r^2 + 608640\Delta r^3 - 76800\Delta r^5 + 12288\Delta r^6 + 12288\Delta r^4)Dnum_3$
H_2	$(-465120r^2\Delta^2 - 195900\Delta^2 r + 128640\Delta^2 r^3 - 15360\Delta^2 r^5 + 714105\Delta^2 + 57600\Delta^2 r^4)Dnum_3$
H_3	$+(-64320\Delta^3 r^2 + 7680\Delta^3 r^4 + 97950\Delta^3)Dnum_3$
Coefficient	**Denominator**
I_0	$(4096r^7 + 30720r^6 + 36096r^5 - 192000r^4 - 454416r^3 + 78360r^2 + 838259r + 506955)/Dnum_3$
I_1	$+(938460\Delta r + 1388142\Delta - 723408\Delta r^2 - 608640\Delta r^3 + 76800\Delta r^5 + 12288\Delta r^6 + 12288\Delta r^4)/Dnum_3$
I_2	$+(-465120r^2\Delta^2 + 195900\Delta^2 r - 128640\Delta^2 r^3 + 15360\Delta^2 r^5 + 714105\Delta^2 + 57600\Delta^2 r^4)/Dnum_3$
I_3	$+(-64320\Delta^3 r^2 + 7680\Delta^3 r^4 + 97950\Delta^3)/Dnum_3$

From the coefficients of Table 1, the 3rd order rational approximation of s^r can be obtained and 3rd order generalized transfer function as given by (20).

$$G_{\delta d}(\gamma) = s^r = \left(\frac{6\gamma + 3\Delta\gamma^2}{6 + 6\gamma\Delta + \Delta^2\gamma^2}\right)^r = \frac{H_0\gamma^3 + H_1\gamma^2 + H_2\gamma + H_3}{I_0\gamma^3 + I_1\gamma^2 + I_2\gamma + I_3} \tag{20}$$

4. DIGITAL REALIZATION OF FRACTIONAL-ORDER $PI^\lambda D^\mu$ CONTROLLER IN THE DELTA DOMAIN

The transfer function of the $PI^\lambda D^\mu$ controller in continuous time is given by (7). To realize the controller transfer functions in the delta domain, fractional order operator such as $s^{-\lambda}$ and s^{μ} are to be implemented in the delta domain using (20).The $PI^\lambda D^\mu$ controller in the delta domain takes the form as

$$C_\delta(\gamma) = K_p + K_i\left(\frac{6\gamma + 3\Delta\gamma^2}{6 + 6\gamma\Delta + \Delta^2\gamma^2}\right)^{-\lambda} + K_d\left(\frac{6\gamma + 3\Delta\gamma^2}{6 + 6\gamma\Delta + \Delta^2\gamma^2}\right)^{\mu} \tag{21}$$

In this work, the proposed FOC, designed in the delta domain is to control a plant, which is of a first order with time delay [33]. The plant transfer function $G_p(s)$ is modeled through the first order Padé approximation to obtain (22).

$$G_p(s) = \frac{k_p}{1 + sT}e^{-Ls} \approx \left(\frac{k_p}{1 + sT}\right)\left(\frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s}\right) \tag{22}$$

Considering $T = 1$, $L = 0.1$, the plant becomes

$$G_p(s) = \frac{k_p}{1 + s}e^{-0.1s} \approx \left(\frac{k_p}{1 + s}\right)\left(\frac{1 - 0.05s}{1 + 0.05s}\right) \tag{23}$$

The FOPID controller in the continuous-time domain is tuned using Particle Swarm Optimization (PSO) [33]for the plant as given by (23) and tuned parameters of the FOPID controllers are as: Proportional gain(K_p) = 0.7469, integral gain(K_i) = 0.874, derivative gain $(K_d) = 0.0001, \lambda = 1.2089$ and $\mu = 0.0603$

The FOPID in discrete delta domain takes the form as shown in (24).

$$C_\delta(\gamma) = 0.7469 + 0.874\left(\frac{6\gamma + 3\Delta\gamma^2}{6 + 6\gamma\Delta + \Delta^2\gamma^2}\right)^{-1.2089} + 0.0001\left(\frac{6\gamma + 3\Delta\gamma^2}{6 + 6\gamma\Delta + \Delta^2\gamma^2}\right)^{0.0603} \tag{24}$$

3rd order rational approximation of the controller in delta domain (sampling time is considered to be $\Delta = 0.001$ second) is obtained using (20) and expressed by (25).

$$\begin{aligned} C_\delta(\gamma) = 0.7469 &+ \left(\frac{-9.514\gamma^3 - 0.0006938\gamma^2 - 1.293e^{-8}\gamma - 7.102e^{-14}}{\gamma^3 - 0.0009524\gamma^2 - 4.021e^{-8}\gamma - 3.909e^{-13}}\right) \\ &+ \left(\frac{9.031e^{-5}\gamma^3 + 1.558e^{-8}\gamma^2 + 4.316e^{-13}\gamma + 3.148e^{-18}}{\gamma^3 + 0.0001543\gamma^2 + 4.106e^{-9}\gamma + 2.911e^{-14}}\right) \end{aligned} \tag{25}$$

4.1. Realization of controller using DF-II method

In this work, the delta domain FOPID controller is realized using Direct Form II (DF-II) realization method.

The FOC can be realized in IIR form in z-domain as follows

$$F(z^{-1}) = \left(\frac{y(z^{-1})}{x(z^{-1})}\right) = \left(\frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}}\right) \tag{26}$$

The FOC can be realized in IIR form in δ-domain as follows:

$$F(\gamma^{-1}) = \left(\frac{y(\gamma^{-1})}{x(\gamma^{-1})}\right) = \left(\frac{m_0 + m_1 \gamma^{-1} + m_2 \gamma^{-2} + \ldots + m_M \gamma^{-M}}{n_0 + n_1 \gamma^{-1} + n_2 \gamma^{-2} + \ldots + n_N \gamma^{-N}}\right) \tag{27}$$

The functional diagram of the Delta DF-II realization method is depicted in Fig.2. corresponding to governing IIR equation (27).

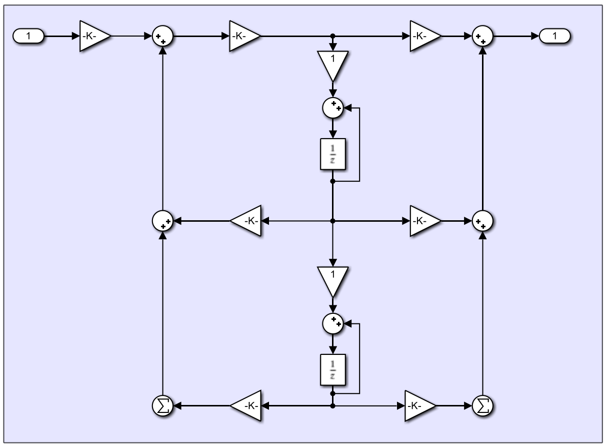


Fig. 2 Delta Direct Form II realization structure

The unit delay block (z^{-1}) corresponding to discrete *z*-domain is rebuilt in the discrete δ-domain using (10) to realize the FOC in delta domain. This can be called as Delta Direct Form -II(DDF-II) realization. The unit delay block (γ^{-1}) in the δ-domain in represented by (28).

$$\gamma^{-1} = \frac{z^{-1}}{\Delta(1 - z^{-1})} \tag{28}$$

4.1.1. Delta Direct Form-II realization of FOI

The integrator part of (25) is considered for the DDF-II realization purpose. In Fig. 3, the DDF-II realization of integrator section is demonstrated.

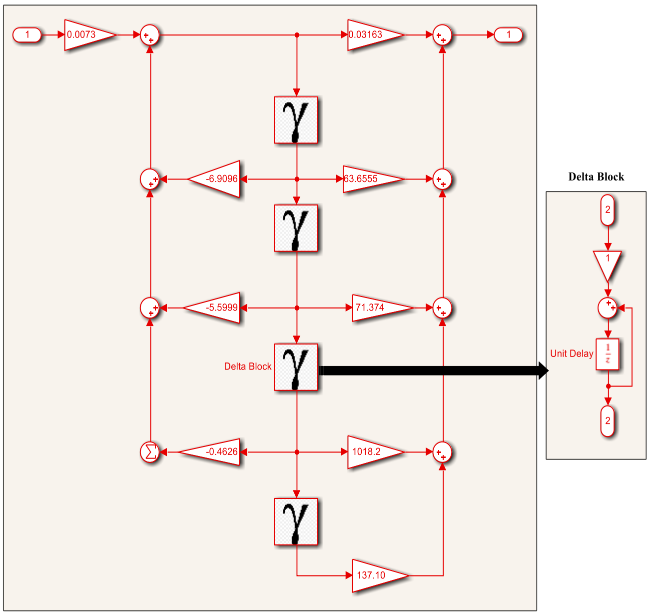


Fig. 3 Delta Direct Form II realization of Fractional-order integrator section of fractional-order controller

4.1.2. Delta Direct Form-II realization of FOD

The differentiator part of (25) is considered for the DDF-II realization purpose. In Fig. 4, the DDF-II realization of differentiator section is demonstrated

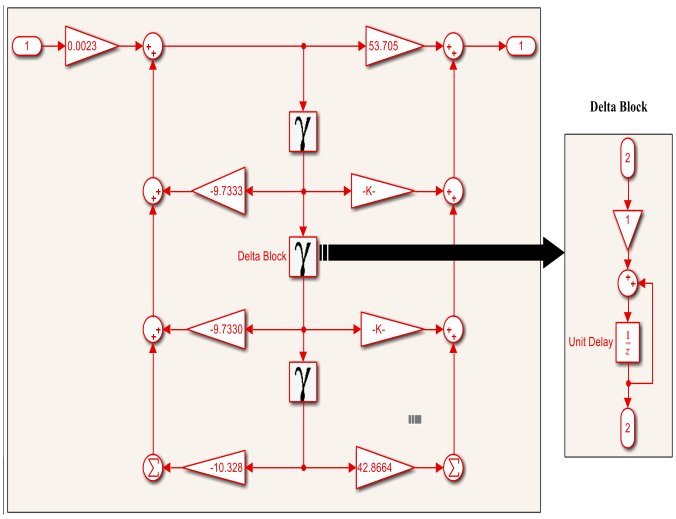


Fig. 4 Delta Direct Form II realization of Fractional-order differentiator section of fractional order controller

4.2. Implementation of digital controller designed in delta domain using dSPACE

Data acquisition and control of the prototype system with a controller is accomplished using DS1202 dSPACEMicroLabBox, which can be reprogrammed using MATLAB/ Simulink, and dSPACE software. The dSPACE is a software package where the real-time interface with the model-based input-output can be integrated with the Simulink control desk.

If any continuous system is to be controlled with a digital controller having a sampling time of Δ, the following functional diagram as shown in Fig. 5can be utilized. The interfacing of the system and the controller can be pictorially demonstrated in Fig.5. To get the information from the sensor to the controller in dSPACE, analog to digital (ADC) converter is used and digital to analog (DAC) is used to send the signal back.

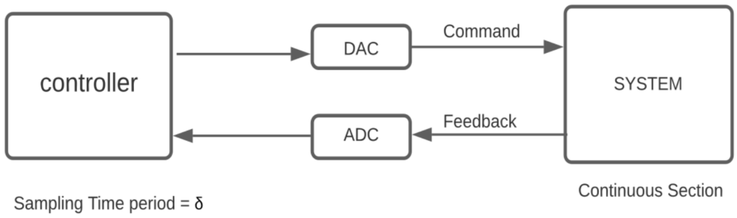


Fig. 5 Real-Time Control Structure

The selection of sample time of the control program using dSPACE depends on the time constant of the physical system, which is again related to the dynamics of the system.

The actual hardware set up for the experiment is shown in Fig.6 where the plant is designed in a continuous-time domain and controller is designed in the delta domain (discrete-time domain) and implemented through the DS1202 dSPACE board.

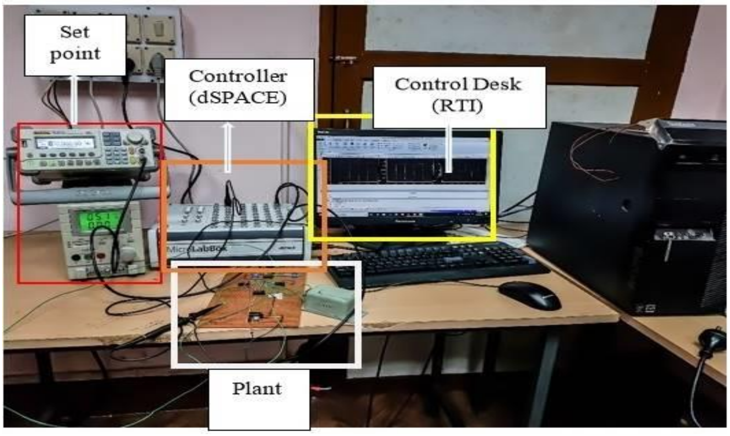


Fig. 6. Actual Photograph of the experimental setup

In Fig. 7, analog realization of FO plant [8] controller in the continuous-time domain is shown. The parameters required to design the FO plant as shown in Fig.7. is summarized in Table 2.

Table 2 Component specifications for designing the FO Plant

Elements	Value
R_1	40 KΩ
R_2	10 KΩ
R_3	500 Ω
C_1, C_2	15 nF

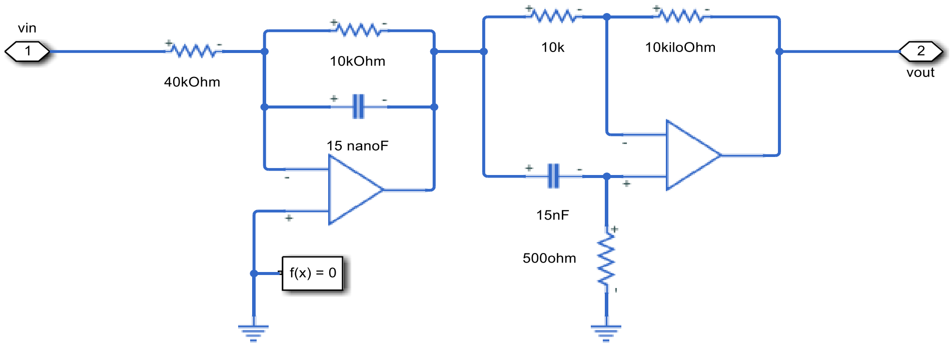


Fig. 7 Analog realization of Fractional order Plant

Fig. 8 shows the digital realization of the FOPID Controller designed using the delta operator used to control the continuous-time plant in MATLAB/Simulink. Fig. 9 demonstrates the step response of the overall system where the FOPID controller using the delta operator is designed using MATLAB/Simulink.

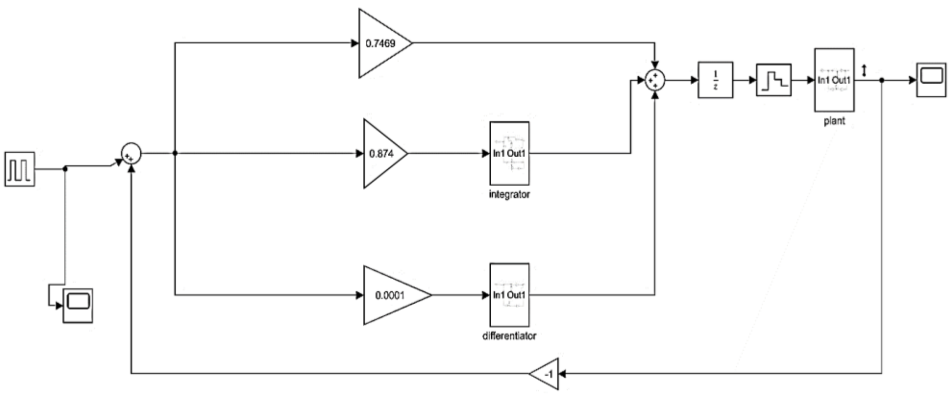


Fig. 8 Digital realization of FOPID controller designed in the delta domain (k_p = 0.25)

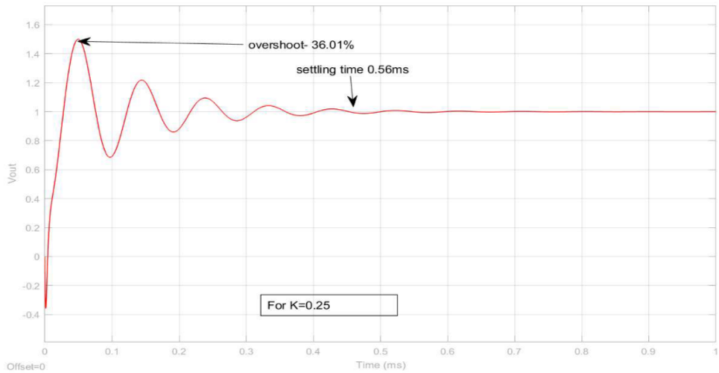
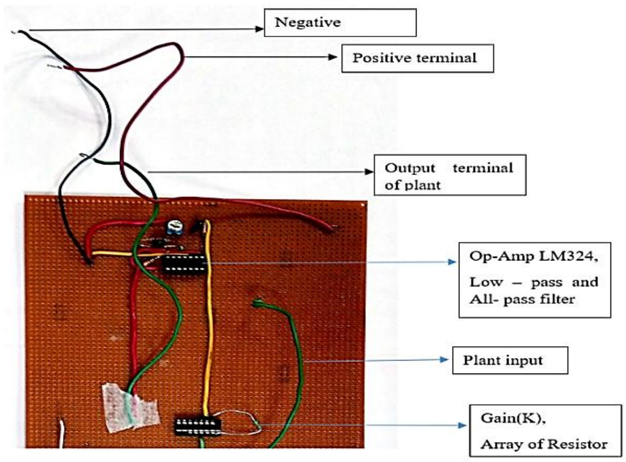


Fig. 9 Step response of the overall system with FOPID controller designed in delta domain



(k_p = 0.25)

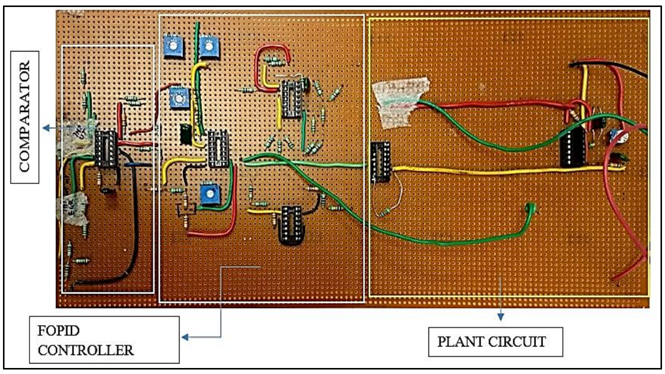


Fig. 10 Hardware implementation of the plant of first order with time delay

5. RESULT ANALYSIS

In this work, delta operator parameterization is used to design the discrete FOPID controller, and the same is realized by Delta Direct Form II structure. The plant is considered to be one first order with time delay, is designed on a real-time basis. The designed delta FOPID controller is implemented using the DS1202 dSPACE board, and the unit step responses of the overall system for variation of the dc gain k_p are demonstrated in Fig. 12 to Fig. 17.

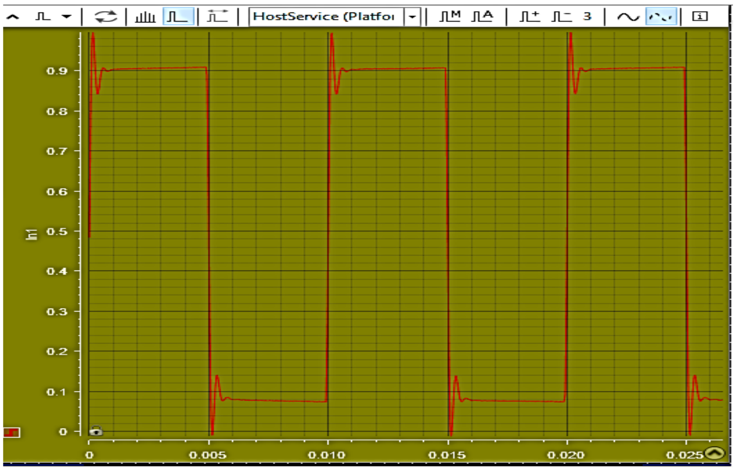


Fig. 12 Step response characteristics of the overall system with delta FOPID controller in dSPACE (k_p = 0.25, the maximum overshoot percentage or M_p (%) = 1.4 and t_s (ms) =1.3)

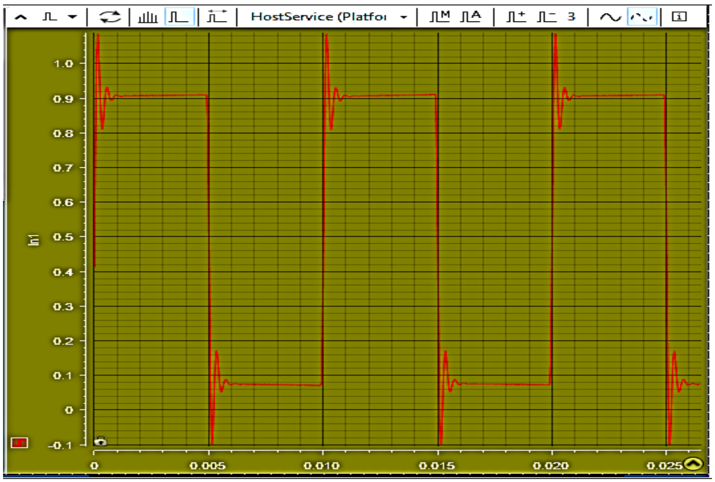


Fig. 13 Step response characteristics of the overall system with delta FOPID controller in dSPACE (k_p = 0.5, the maximum overshoot percentage or M_p (%) = 9.28 and t_s (ms) = 1.5)

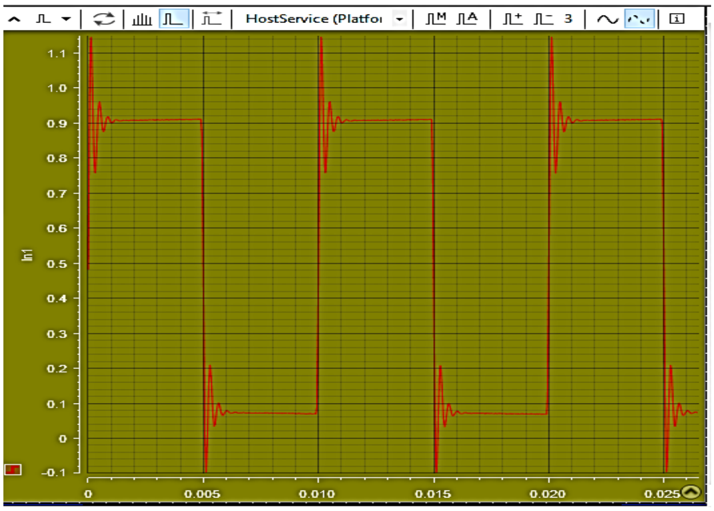


Fig. 14 Step response characteristics of the overall system with delta FOPID controller in dSPACE ($k_p = 1$, the maximum overshoot percentage or M_p (%) = 14.53 and t_s (ms) = 1.6)

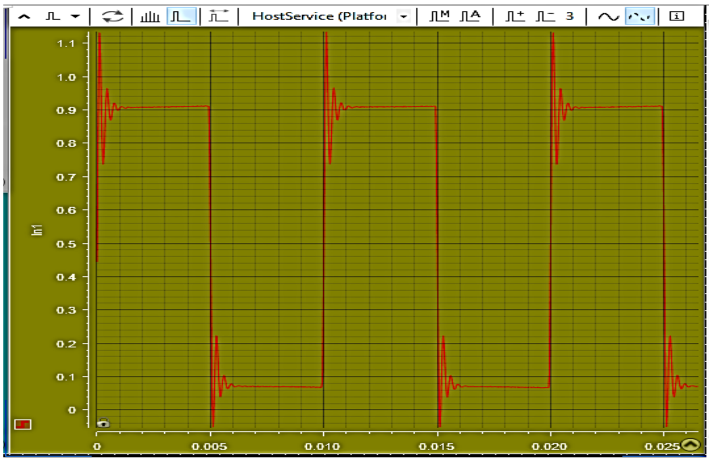


Fig. 15 Response characteristics of the overall system with delta FOPID controller in dSPACE (k_p = 2, the maximum overshoot percentage or M_p (%) = 13.59 and t_s (ms) = 1.2)

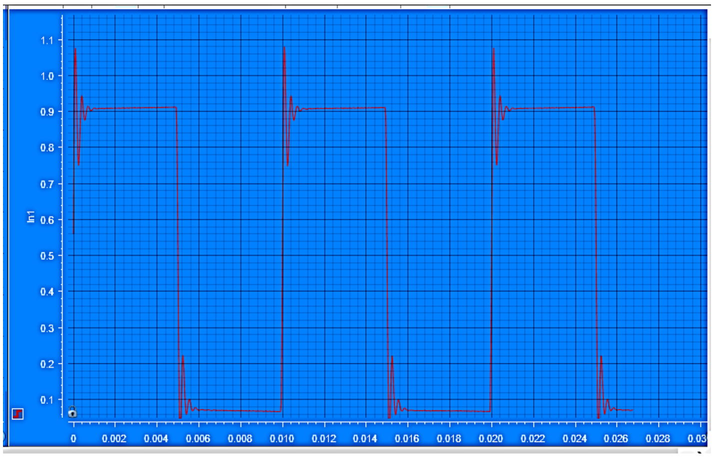


Fig. 16 Step response characteristics of the overall system with delta FOPID controller in dSPACE ($k_p = 4$, the maximum overshoot percentage or M_p (%) = 7.15 and t_s (ms) = 1.14)

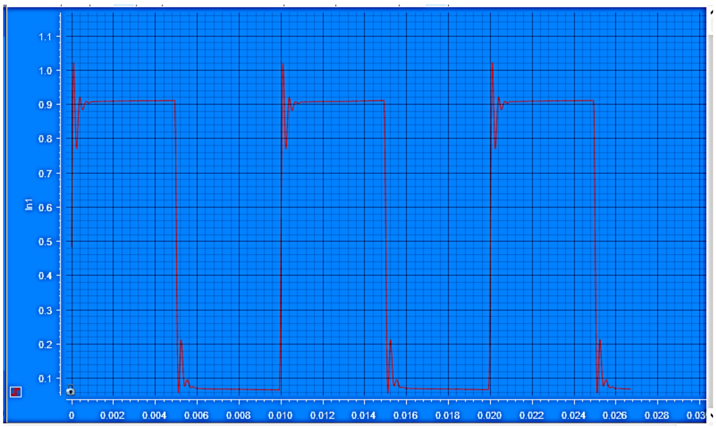


Fig. 17 Step response characteristics of the overall system with delta FOPID controller in dSPACE ($k_p = 8$, the maximum overshoot percentage or M_p (%) = 2.309 and t_s (ms) = 0.96)

5.1. Robustness analysis for the proposed controller

To study the robustness analysis of the developed delta domain FOC, the dc gain (k_p) is varied and the responses of the closed loop system are measured. For the variation of dc gain (k_p), the peak percentage overshoot and the settling time are measured, and variation of the percentage peak overshoot and settling times does not vary considerably for the variation of dc-gain. The iso-damping property of fractional-order system is thus satisfied through the designing of discrete FOC in delta domain. A comparative analysis of the time domain parameters for variation of the dc gain (k_p) has been summarized in Table 3.

From the plots shown in Fig. 12 to Fig. 17, proves that the closed loop system with delta FOPID controller realized using dSPACE is robust against process gain (k) variations and exhibits the iso-damping properties.

5.2 Sensitivity analysis of the system

A perturbation (± 20 % *pu*) is applied to the closed loop system containing the fractional order plant and the developed delta domain FOPID using dSPACE and the steady state response in noted. The output of the closed loop system with random variation of step input, is demonstrated in Fig. 18. From the Fig. 18, it is very clear that the steady state error becomes zero though a sufficient perturbation is applied at the input side. This proves the system to be a robust one and sensitive to input variation .

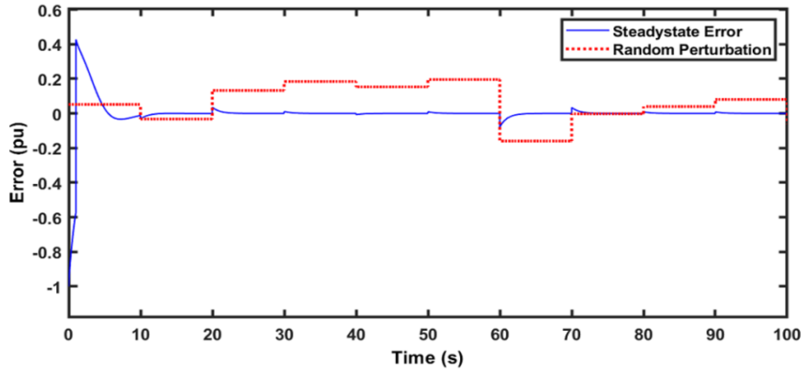


Fig. 18 Steady state error of the closed loop system for a random perturbation

The FOC designed using continuous and discrete delta domain must have to be stable. The pole -zero plotting of the designed controller in both domains are shown in Fig. 19 and Fig. 20. From Fig. 19 and Fig. 20 the stability of the realized controllers is ensured.

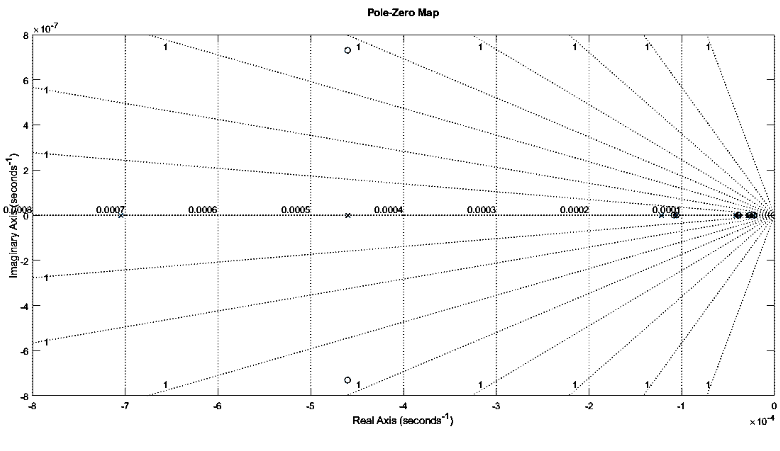


Fig. 19 Pole-Zero Plot of discrete delta(γ) FOPID Controller

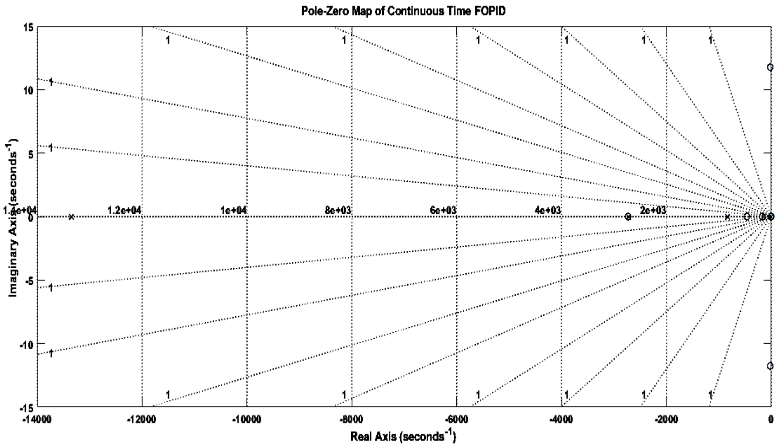


Fig. 20 Pole-Zero Plot of continuous time FOPID Controller

Table 3 Comparative analysis of the time domain parameters for variation of the dc gain 'k_p'

Realization methods		S-Domain realization	Analog realization [33]	Delta Domain realization
k_p = 0.25	$\%M_P$	11.2	4.11	1.4
	$t_S\,(ms)$	0.86	0.54	1.1
k_p = 0.5	$\%M_P$	12.9	10.9	9.28
	$t_S\,(ms)$	0.52	0.32	.95
k_p = 1	$\%M_P$	14.23	14	14.53
	$t_S\,(ms)$	0.29	0.2	.74
k_p = 2	$\%M_P$	11.29	12.3	12.59
	$t_S\,(ms)$	0.17	0.11	1.1
k_p = 4	$\%M_P$	7.3	7.9	7.1
	$t_S\,(ms)$	0.07	0.052	1.14
k_p = 8	$\%M_P$	8.1	5.8	2.3
	$t_S\,(ms)$	0.021	0.017	0.96

6. CONCLUSION

In this paper, the design and implementation of fractional order controller in the delta domain is presented. One of the essential properties of the fractional-order system is iso-damping property. The fractional-order PID controller is designed in delta domain from corresponding continuous-time FOPID controller transfer function by using the direct discretization method and the delta FOPID controller is then realized using delta direct form-II structure of filter realization. The DS1202 dSPACE board is used in this work to implement the controller through the MATLAB/Simulink and control desk interface of the dSPACE board. This approach is devoid of ill-conditioning which is inherentin the case with shift operator parameterization. In this work, the sampling rate (Δ=0.001 sec) is considered very close to zero to obtain a discrete time system with very high sampling rate. The FOPID controller designed in the delta domain gives the response characteristics very close to the responses obtained from the analog realization of the FOPID controller, which is designed in the S-domain. When the dc gain "k_p" is varied over a specified range, the response characteristics of the overall system remains almost unaltered meaning the property of iso-damping is satisfied. From the Table 3, it is evident that the results are very close to each other in regard to the time response parameters among the three methods of designing FOPID controller. The stability of the realized system is also verified through the pole and zero locations of developed delta domain controller. The system response remains stable with a perturbation in the step input as demonstrated in Fig.18.The results obtained using delta parameterized discrete-time system resembles to that of the results as obtained by continuous-time system at a fast-sampling rate makes the design a unified one and a viable alternative for the discrete fractional order controller design and implementation.

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