



# Control of a nonlinear coupled roll-pitch ship using time-delayed state feedback

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**Abstract.** The work scripted in this paper shows the effectiveness of using time-delayed state feedback to control the vibration of nonlinear coupled roll-pitch motion of a two-degree-of-freedom (2 DOF) system under harmonic excitation. At first, the uncontrolled frequency responses graphs are obtained under various non-resonance and resonance cases, including sub-harmonics and chaos. Next, the same system is investigated for vibration control by time-delayed feedback. The appropriate sets of control gain and delay are selected from the linear stability study of the corresponding linear oscillator. An analytical approach, namely, the Slowly Varying Parameter (SVP) method, is employed for this purpose. Results obtained by the SVP method are validated by the numerical integration (NI) technique at discrete points and they are found to be good in agreement. Moreover, it is seen that using proper control parameters, the nonlinear resonance curves of primary, secondary, sub-harmonic, simultaneous resonances and chaotic behavior of the system during rolling and pitching can be suppressed considerably, resulting in almost linear frequency responses.

**Keywords.** Nonlinear coupled roll-pitch; period one solution; resonance; sub-harmonic; stability; state feedback.

## 1. Introduction

A ship has six degrees of freedom (DOF), three linear motions (surge, sway/drift, and heave) and three rotational motions (roll, pitch and yaw) along and about longitudinal, transverse and vertical directions. Out of these six motions, the rolling of a ship and the roll-pitch coupling plays a major and critical role in the capsizing of the ship. Steadiness against capsizing in severe oceanic environmental conditions is one of the prime research areas. Moreover, as the amplitudes of oscillation along with the ocean wave load increase, the nonlinearities come into play. These arise due to the nature of restoring moment, environmental loading, damping etc. In this paper, a study of the response of the nonlinear coupled roll-pitch system under state feedback is presented, which is based on the various system parameters' values reported in the literature [1–4].

Nayfeh and co-workers have studied the motion of modulated sinusoidal excited nonlinearly coupled roll-pitch ship [2, 5, 6]. The investigation has been carried out for different motions of a ship [7–9] to get the response of a 2DOF system with quadratic coupling under a sinusoidally

modulated amplitude excitation. In the motion of the ship, due to nonlinearity, a wide range of phenomena like jump, multiple solutions, period multiplying bifurcation, frequency entrapment, dynamic chaos and different resonance cases such as primary, internal, combination, simultaneous and sub-harmonic resonances, which occur frequently, are discussed in the literature [10–14]. Dynamic responses of coupled roll and pitch modes of a ship model with nonlinearity under modulated excitation have been studied by Pan with his co-workers [1, 3, 15]. Different techniques like the multiple time scale method and averaging method are used to analyze the effect of the modulation on the performance of the system. Various cases like local and global stability, periodic response and chaotic behavior of the system were reported after detailed discussion [3, 16]. Regarding control of nonlinear resonances, Hu *et al* [17] has discussed thoroughly about the effect of time delay on the primary and sub-harmonic resonances by considering time-delayed state feedback control of a harmonically forced Duffing oscillator with cubic restoring force features. It is observed that a proper selection of the feedback gains and time delay can enhance control performance by eliminating saddle-node bifurcation, thereby eliminating jump and hysteresis phenomena. Mitra, with his co-workers [18], has discussed the region of gain and delay values of

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