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A Fractional-Order Transitional Butterworth-Butterworth Filter and Its Experimental Validation

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ABSTRACT This paper introduces the generalization of the classical Transitional Butterworth-Butterworth Filter (TBBF) to the Fractional-Order (FO) domain. Stable rational approximants of the FO-TBBF are optimally realized. Several design examples demonstrate the robustness and modeling efficacy of the proposed method. Practical circuit implementation using the current feedback operational amplifier employed as an active element is presented. Experimental results endorse good agreement ($R^2 = 0.999968$) with the theoretical magnitude-frequency characteristic.

INDEX TERMS Analog filter approximation, analog signal processing, current feedback operational amplifier, fractional-order filter, transitional filter.

I. INTRODUCTION

The modeling techniques and realization of classical (integer-order) analog filters are well-established. To further improve the performance of such filters (e.g., reduction in pass-band error, sharper transition-band characteristic), the use of graphical methods [1] and optimal procedures [2]–[4] have been adopted.

Recently, the theoretical concept of fractional calculus, which deals with the generalization of the classical definitions of differentiation and integration, has been applied to achieve a more precise attenuation behavior of analog filters [5]. This is possible due to the generalization of the classical Laplacian operator s to the Fractional-Order (FO) form s^α , where $\alpha \in (0, 1)$, which causes additional degrees of freedom in system modeling. The impedance function containing the s^α operator may be realized using fractance devices or Constant Phase Elements (CPE) [6]. Due to the commercial unavailability of these devices, CPE emulators in the integrated form [7] or discrete-components-based [8] have been reported. The s^α operator forms the basic building

block of the FO transfer functions, which can lead to generalizations of classical Butterworth filter [9], oscillators [10], and resonators [11]. Both active and passive elements have been employed to realize the FO impedances [12], [13]. Another popular method is to approximate the FO system using the integer-order transfer function [14]. The exact dynamics of a FO system can be theoretically achieved by a system of infinite integer order. For practical purposes, the characteristics of the FO filter need to be approximated using a finite-order rational approximant. An integer-order model of lower-order is desirable since it results in smaller hardware overhead. The rational approximation of s^α may be achieved using frequency-domain-based curve fitting [15], a weighted sum of first-order optimal high-pass filter sections [16], etc.

Transitional filters merge the frequency responses of various classical filters (e.g., Butterworth, Chebyshev, Bessel, Legendre, Thomson) to attain conciliation between the amplitude and group delay characteristics [17]. Transitional filters may be designed by combining different filter poles using the arithmetic or geometric interpolation, as exemplified by the transitional Legendre-Thomson filter [18], and the transitional ultraspherical-ultraspherical filter [19]. An alternative

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fractional-order analog and digital filters. He was awarded with the University Medal for his M.Tech. degree from Jadavpur University. He was a recipient of the 2019 Premium Award for Best Paper from *IET Signal Processing*.



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