fractal and fractional

Article

Further Generalization and Approximation of Fractional-Order Filters and Their Inverse Functions of the Second-Order Limiting Form

Shibendu Mahata [1], **Norbert Herencsar** [2,*] and **David Kubanek** [2]

[1] Department of Electrical Engineering, Dr. B. C. Roy Engineering College,
Durgapur 713206, West Bengal, India; shibendu.mahata@bcrec.ac.in
[2] Department of Telecommunications, Faculty of Electrical Engineering and Communication,
Brno University of Technology, Technicka 12, 61600 Brno, Czech Republic; kubanek@vutbr.cz
* Correspondence: herencsn@ieee.org; Tel.: +420-541-146-981

Abstract: This paper proposes a further generalization of the fractional-order filters whose limiting form is that of the second-order filter. This new filter class can also be regarded as a superset of the recently reported power-law filters. An optimal approach incorporating constraints that restricts the real part of the roots of the numerator and denominator polynomials of the proposed rational approximant to negative values is formulated. Consequently, stable inverse filter characteristics can also be achieved using the suggested method. Accuracy of the proposed low-pass, high-pass, band-pass, and band-stop filters for various combinations of design parameters is evaluated using the absolute relative magnitude/phase error metrics. Current feedback operational amplifier-based circuit simulations validate the efficacy of the four types of designed filters and their inverse functions. Experimental results for the frequency and time-domain performances of the proposed fractional-order band-pass filter and its inverse counterpart are also presented.

Keywords: analog filter approximation; current feedback operational amplifier; fractional-order filter; inverse filter; optimization; power-law filter; second-order filter

check for updates

Citation: Mahata, S.; Herencsar, N.; Kubanek, D. Further Generalization and Approximation of Fractional-Order Filters and Their Inverse Functions of the Second-Order Limiting Form. *Fractal Fract.* **2022**, *6*, 209. https://doi.org/10.3390/fractalfract6040209

Academic Editor: Da-Yan Liu

Received: 10 February 2022
Accepted: 5 April 2022
Published: 8 April 2022

1. Introduction

The concepts of fractional calculus [1], the branch of mathematics which generalizes the integration and differentiation operations, have seen widespread applications in various fields of science and engineering [2]. The Grunwald–Letnikov definition of a fractional derivative of order α for a function $f(t)$ is given by (1) [3].

$$ {}_aD_t{}^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \begin{pmatrix} \alpha \\ j \end{pmatrix} f(t-jh) \quad (1)$$

where $[x]$ denotes the integer part of x; $\begin{pmatrix} \alpha \\ j \end{pmatrix} = \frac{\Gamma(\alpha+1)}{\Gamma(j+1)\Gamma(\alpha-j+1)}$ represents the binomial coefficients; a and t are the bounds of the operation; and $\alpha \in (0,1)$. Under zero initial conditions, the Laplace transformation of (1) is given by (2).

$$L\{{}_0D_t^{\alpha} f(t)\} = s^{\alpha} F(s) \quad (2)$$

The presence of the additional tuning parameter α provides several fundamental advantages to fractional-order (FO) filters when compared against the traditional (integer-order) filters: (i) exact meeting of design specifications, which implies precise control of filter roll-off characteristics. For instance, the fractional-order low-pass filter (FLPF) exhibits