fractal and fractional

Article

On the Design of Power Law Filters and Their Inverse Counterparts

Shibendu Mahata [1], **Norbert Herencsar** [2,*] and **David Kubanek** [2]

[1] Department of Electrical Engineering, Dr. B. C. Roy Engineering College, Durgapur 713206, PO, India; shibendu.mahata@bcrec.ac.in

[2] Department of Telecommunications, Faculty of Electrical Engineering and Communication, Brno University of Technology, Technicka 12, 61600 Brno, Czech Republic; kubanek@vutbr.cz

* Correspondence: herencsn@ieee.org; Tel.: +420-541-146-981

Abstract: This paper presents the optimal modeling of Power Law Filters (PLFs) with the low-pass (LP), high-pass (HP), band-pass (BP), and band-stop (BS) responses by means of rational approximants. The optimization is performed for three different objective functions and second-order filter mother functions. The formulated design constraints help avoid placement of the zeros and poles on the right-half s-plane, thus, yielding stable PLF and inverse PLF (IPLF) models. The performances of the approximants exhibiting the fractional-step magnitude and phase responses are evaluated using various statistical indices. At the cost of higher computational complexity, the proposed approach achieved improved accuracy with guaranteed stability when compared to the published literature. The four types of optimal PLFs and IPLFs with an exponent α of 0.5 are implemented using the follow-the-leader feedback topology employing AD844AN current feedback operational amplifiers. The experimental results demonstrate that the Total Harmonic Distortion achieved for all the practical PLF and IPLF circuits was equal or lower than 0.21%, whereas the Spurious-Free Dynamic Range also exceeded 57.23 and 54.72 dBc, respectively.

Keywords: analog filter approximation; analog signal processing; fractional-order filter; inverse filter

Citation: Mahata, S.; Herencsar, N.; Kubanek, D. On the Design of Power Law Filters and Their Inverse Counterparts. *Fractal Fract.* **2021**, *5*, 197. https://doi.org/10.3390/fractalfract5040197

Academic Editor: Ricardo Almeida

Received: 21 September 2021
Accepted: 1 November 2021
Published: 4 November 2021

1. Introduction

The theoretical concepts of fractional calculus [1–3], which generalized differ-integral operators, have led to significant developments in circuit theory, signal processing, control theory, bio-impedance modeling, etc. [4–8]. Fractional-order (FO) filters are considered as the generalization of the traditional filters [9]. This is due to the ability of the FO filters to achieve any roll-off rate [10]; in contrast, an integer-order filter can only achieve a roll-off at $-20\log_{10} n$ decibels/decade (dB/dec), where n is an integer [11]. FO analog filter transfer functions are generally realized from the integer-order filters by substitution of the Laplacian operator s with the non-integer Laplacian operator s^α, where $\alpha \in (0,\ 1)$. The frequency–domain transfer function of s^α is given by (1):

$$(j\omega)^\alpha = \omega^\alpha \left[\cos\left(\frac{\alpha\pi}{2}\right) + j\sin\left(\frac{\alpha\pi}{2}\right)\right], \tag{1}$$

where $j = \sqrt{-1}$ and ω is the angular frequency in radians per second (rad/s).

Since s^α is an irrational function, various rational approximations based on series truncation, frequency–domain curve-fitting, pole-zero placement, optimization techniques, etc., have been reported [12–15]. The impedance characteristics of the operator s^α may be practically realized using the FO elements (also known as the fractance devices or the constant phase elements) [16–18]. Due to the unavailability of the commercial FO device, their behavior may be emulated using the passive and active circuits [19–22].

Recent works have demonstrated the generalization of the Butterworth [23], Chebyshev [24], inverse Chebyshev [25], and elliptic filters [26] to the FO domain. Another design