## ECA-MURE Algorithm and CRB Analysis for High-Precision DOA Estimation in Coprime Sensor Arrays

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Abstract—The enhanced coprime array (ECA), in conjunction with the manifold reconstruction unitary ESPRIT (MURE) algorithm, offers a groundbreaking solution for direction-of-arrival (DOA) estimation in coprime sensor arrays. Coprime arrays, known for their ability to capture signals from multiple uncorrelated sources, present unique challenges due to their nonstandard geometry. The MURE algorithm seamlessly combines manifold reconstruction and unitary ESPRIT techniques to address these challenges. One significant contribution of this research is the Cramér—Rao Bound (CRB) analysis, shedding light on the fundamental limits of DOA estimation in coprime arrays. In addition, a comprehensive computational complexity analysis provides insights into the algorithm's efficiency. Extensive computer simulations consistently demonstrate the superior performance of the ECA-MURE algorithm, showcasing its accuracy and robustness. This innovative approach has far-reaching implications for applications relying on precise DOA estimation, including radar systems, wireless communication, and acoustic sensing. ECA-MURE unlocks the potential of coprime arrays, making them more practical and effective in real-world scenarios.

Index Terms—Sensor systems, coprime array, direction-of-arrival (DOA) estimation, manifold reconstruction, sensor array, signal processing, unitary ESPRIT.

## I. INTRODUCTION

Significant advancements in direction-of-arrival (DOA) estimate techniques have been made in sensor array signal processing over the last 20 years, primarily through the use of the parametric approach, which has improved signal source localization and tracking [1]. However, these advancements have unveiled new challenges, particularly when traditional sensor arrays fall short. A groundbreaking development in DOA estimation is the emergence of coprime array technology, known for its ingenious sensor placement. Coprime arrays excel in capturing signals from multiple uncorrelated sources with remarkable precision. Yet, their unconventional geometry introduces complexity, necessitating innovative DOA estimation solutions. Coprime arrays significantly increase degrees of freedom for beamforming and DOA estimation, allowing denser signal autocorrelation estimation compared with sparse sensor spacing, resulting in more precise estimates [1]. Numerous researchers have contributed to the field, such as those in [2] and [3], who focus on narrowband source DOA estimation using coprime arrays through atomic norm minimization, improving robustness and accuracy.

Zheng et al. [4] introduced symmetric displaced coprime array configurations in, excelling in localizing sources in both near and far fields, highlighting coprime arrays' versatility. In addition, Zhang et al. [5] presented coprime array-based sparsity-based DOA estimation methods in, utilizing signal source sparsity for increased accuracy.

Corresponding author: Veerendra D (e-mail: veerendra@ieee.org) Associate Editor: F. Falcone. Digital Object Identifier 10.1109/LSENS.2023.3332673 Contributions by Tan et al. [6], Shen et al. [7], and Shi et al. [8] introduced super-resolution DOA estimation, low-complexity DOA estimation for wideband coprime arrays, and insights into source estimation from a sparse reconstruction perspective, respectively. An ambiguity-free method of DOA estimation was presented by Zheng et al. [9] in response to ambiguity issues in coprime linear arrays. In this work, we present enhanced coprime array (ECA) manifold reconstruction unitary ESPRIT (MURE), an extended iteration of [10], and a specialized DOA estimation technique tailored for coprime arrays, coupled with comprehensive Cramér–Rao Bound (CRB) analysis.

## II. EXTENDED COPRIME ARRAY SIGNAL MODEL

We analyze coprime array signal model in the following lines. The extended coprime array, formed by unique coprime integers M and N, blends two sparsely spaced uniform linear subarrays. For simplicity, we assume M < N. Illustrated in Fig. 1, one subarray comprises 2M sensors spaced by Nd, while the other features N sensors with Md spacing.

The sensor spacing unit d is half a wavelength, i.e.,  $d = \frac{\lambda}{2}$ . The extended coprime array comprises |S| = 2M + N - 1 sensors at strategic positions, derived from the set S [11]:

$$S = \{Mn | 0 < n < N-1\} \cup \{Nm | 0 < m < 2M-1\}.$$

With K uncorrelated far-field narrowband sources arriving at the array with directions  $\theta = [\theta_1, \theta_2, \dots, \theta_K]^T$ , the array records T

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